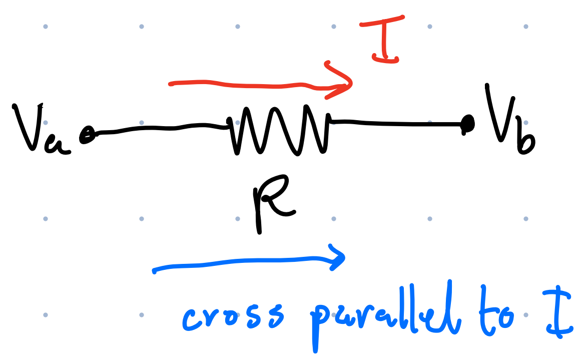
PHYS 121 March 10, 2025

- Complete PrairieLearn HW by 23:59 on March 14.
- Complete Pre-Lab #6 before the start of your lab.

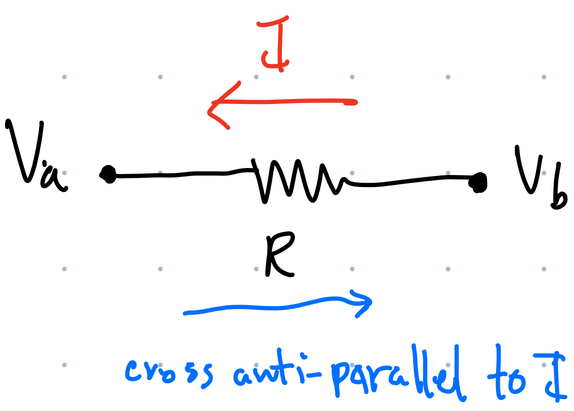
- Quiz #2 will be March 19. See the course website for details / formula sheet.

Cut off for Quiz #2 : Up to & including the matrial from the March 7 class.

Last Time:



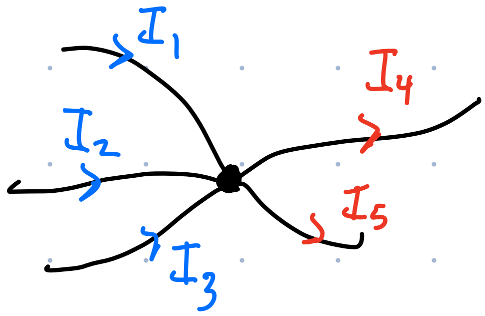
$$\Delta V_R = V_b - V_a$$
$$= -IR$$



$$\Delta V_R = V_b - V_a$$
$$= +IR$$

Previously: Kirchhoff Laws

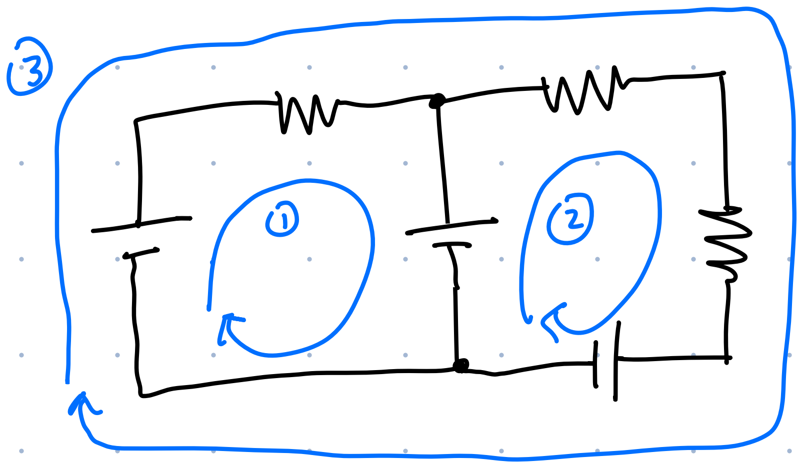
1. Junction Rule



Conservation of charge:

current into jcn = current out of jcn

2. Loop Rule

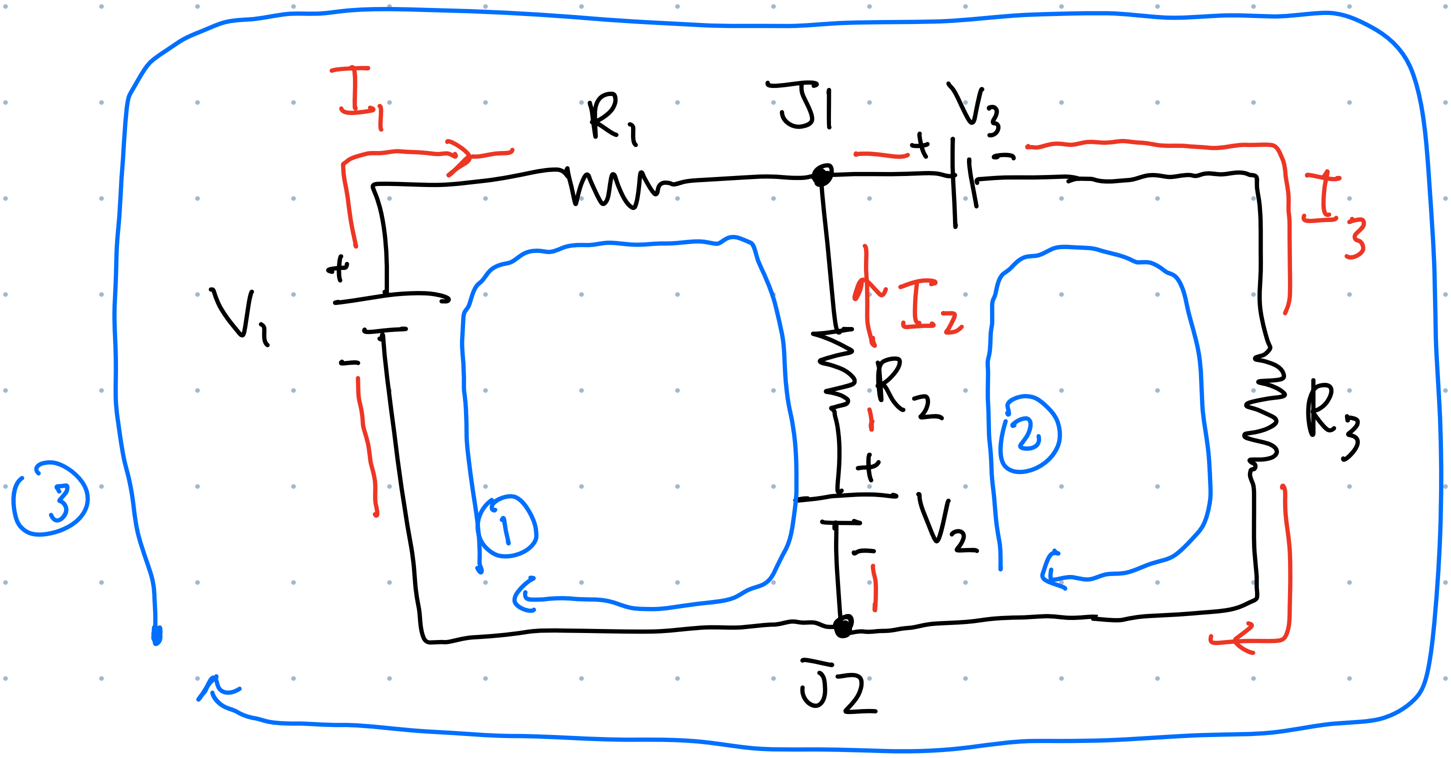


Conservation of energy:

Net change in voltage around any closed loop in a circuit is zero.

$$\sum_{\substack{i \\ \text{loop}}} \Delta V_i = 0$$

Eg. Consider the following circuit consisting of batteries and resistors



Typically in a circuit like this, we will have some unknowns to solve for (currents, battery voltages, resistor values).

Step up a system of N eq'ns to solve for N unknowns.

Eg. In the circuit above find the current in each branch of the circuit (current in each resistor).

Step 1: Assign a current to each branch of the circuit. Label the currents & pick a dir'n.

If at the end of the calc. we find a negative value for a current, it just means that we chose the wrong dir'n. The value of the current (assuming no mistakes) will be correct.

Step 2: Identify the unknowns.

In this example, assume battery voltages & resistor values are given.

Unknowns are: I_1, I_2, I_3.

To solve for the 3 unknowns, need to construct a system of 3 indep. equations involving the 3 unknowns.

Step 3: Apply the junction to start building our system of eq'ns.

Eg. J1: $I_{in} = I_{out}$

$$\boxed{I_1 + I_2 = I_3} \quad (a)$$

J2: $I_3 = I_1 + I_2$ } same expression obtained from J1.

→ no new info.

Step 4: Use the loop rule to find the other two required eq'ns.

Loop ①: $$0 = \underbrace{+V_1}_{\text{neg-to-pos}} \overbrace{-I_1 R_1}^{\text{parallel to } I_1} \underbrace{+I_2 R_2}_{\text{anti-parallel to } I_2} \underbrace{-V_2}_{\text{pos-to-neg.}}$$

Loop ②: $$0 = +V_2 - I_2 R_2 - V_3 - I_3 R_3$$

Loop ③: $$0 = +V_1 - I_1 R_1 - V_3 - I_3 R_3$$

3 loop Eq'ns. Notice that there are only 2 indep. eq'ns.

Consider (loop 1) + (loop 2)

$$= \left(V_1 - I_1 R_1 + \cancel{I_2 R_2} - \cancel{V_2}\right)$$
$$+ \left(\cancel{V_2} - \cancel{I_2 R_2} - V_3 - I_3 R_3\right)$$
$$= V_1 - I_1 R_1 - V_3 - I_3 R_3 = \text{loop } 3$$

Since we can construct the 3rd eq'n using the first two, only have two indep. loop eq'ns.

Step 5: Finalize system of 3 eq'ns
→ use one jcn rule
→ any two of the loop rules.

For our current example:

$$I_1 + I_2 = I_3 \quad (a)$$
$$V_1 - I_1 R_1 + I_2 R_2 - V_2 = 0 \quad (b) \rightarrow \text{loop } 1$$
$$V_2 - I_2 R_2 - V_3 - I_3 R_3 = 0 \quad (c) \rightarrow \text{loop } 2$$

Step 6: Solve the system of 3 eq'ns for the 3 unknowns (just a math problem)

Eg. Take the circuit drawn above and assume

$$V_1 = V_0 \qquad R_1 = R_0$$
$$V_2 = 2V_0 \qquad R_2 = 2R_0$$
$$V_3 = 3V_0 \qquad R_3 = R_0$$

System of 3 eq'ns becomes:

$$I_1 + I_2 = I_3 \quad (a)$$

$$V_0 - I_1 R_0 + 2I_2 R_0 - 2V_0 = 0$$
$$-V_0 - I_1 R_0 + 2I_2 R_0 = 0$$
$$\therefore -\frac{V_0}{R_0} - I_1 + 2I_2 = 0 \quad (b)$$

$$V_2 - I_2 R_2 - V_3 - I_3 R_3 = 0$$
$$2V_0 - 2I_2 R_0 - 3V_0 - I_3 R_0 = 0$$
$$-V_0 - 2I_2 R_0 - I_3 R_0 = 0$$
$$\therefore -\frac{V_0}{R_0} - 2I_2 - I_3 = 0 \quad (c)$$

To solve for the unknowns, start by using two of the eq'ns to eliminate one of the unknowns.

Sub ⓐ into ⓒ to eliminate I_3

$$-\frac{V_0}{R_0} - 2I_2 - (I_1 + I_2) = 0$$

$$\therefore -\frac{V_0}{R_0} - I_1 - 3I_2 = 0 \qquad \text{ⓓ}$$

Now, eq'ns ⓑ & ⓓ form a system of two eq'ns & two unknowns.

Take ⓑ - ⓓ to eliminate I_1

$$\left(\cancel{-\frac{V_0}{R_0}} \cancel{- I_1} + 2I_2\right) - \left(\cancel{-\frac{V_0}{R_0}} \cancel{- I_1} - 3I_2\right) = 0$$

$$\therefore \quad 2I_2 + 3I_2 = 0$$

$$\therefore 5I_2 = 0 \implies \boxed{I_2 = 0}$$

Sub known value of I_2 into (b) or (d) to find I_1

(b) $$-\frac{V_0}{R_0} - I_1 + \cancelto{0}{2\overset{I_2}{(0)}} = 0$$

$$\boxed{\therefore I_1 = -\frac{V_0}{R_0}}$$

neg., so in opp dir'n as drawn.

Sub known values of I_1 & I_2 into (a) to find I_3

$$I_1 + I_2 = I_3$$

$$-\frac{V_0}{R_0} + 0 = I_3$$

$$\boxed{\therefore I_3 = -\frac{V_0}{R_0}}$$

in dir'n <u>opp.</u> to what was drawn.