PHYS 121 March 7, 2025

- Complete PrairieLearn HW by 23:59 $\boxed{\text{today}}$
- Complete Pre-Lab #6 before the start of your lab.

<u>Last Time:</u> $I = env_d A$

$$J = \frac{I}{A} = env_d$$

Ohmic materials obey: $J = \sigma E$

$$\Downarrow$$

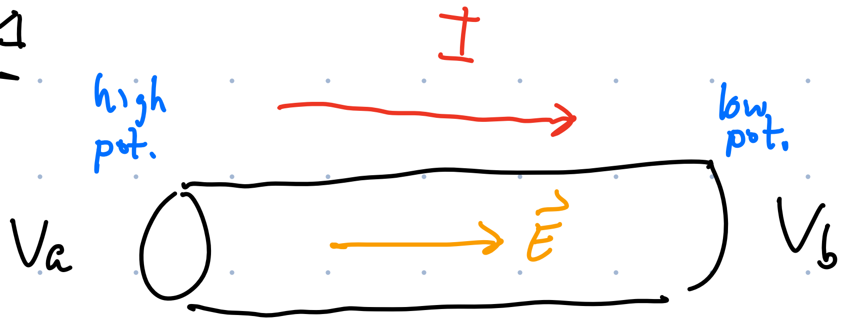
$$\Delta V = IR$$

where $R = \rho \frac{l}{A} = \frac{1}{\sigma}\frac{l}{A}$

<u>Today:</u> When is ΔV across a resistor positive & when is it negative?

<u>Rule of Thumb:</u>
- If you cross a resistor in the dir'n of the current I, the voltage change is negative.
- If you cross a resistor in opp. dir'n of current I, the voltage change is positive.

CASE 1



cross resistor in dir'n of I →

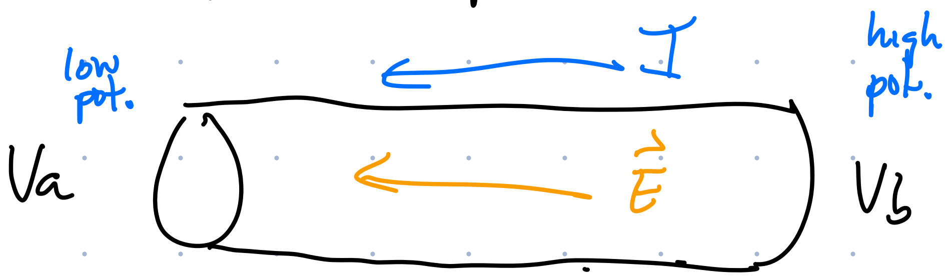
Track changes in voltage.

$$V_a + \underbrace{\Delta V_R}_{\substack{\text{change in} \\ \text{voltage across} \\ \text{resistor}}} = V_b$$

$$\therefore \Delta V_R = \overset{\text{low pot}}{V_b} - \overset{\text{high pot}}{V_a} < 0$$

$$\boxed{\begin{array}{c} \therefore \text{ when cross } R \\ \text{in dir'n of } I \\ \Delta V_R = -IR \end{array}}$$

CASE 2 Cross R antiparallel to I



cross R in opp. dir'n of I →

$$V_a + \Delta V_R = V_b$$

$$\therefore \Delta V_R = \underset{\text{high pot.}}{V_b} - \underset{\text{low pot.}}{V_a} < 0$$

$$\boxed{\begin{array}{c}\text{when across } R \\ \text{antiparallel to } I \\ \Delta V_R = +IR\end{array}}$$

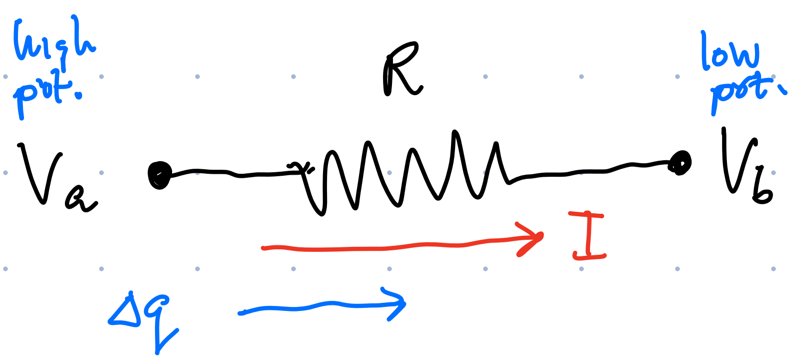
Power Dissipated by a Restor (OSUPv2 sec. 9.5)

Recall that power is defined as:

$$P = \frac{\text{change in energy}}{\text{time interval}}$$

$$[P] = \frac{J}{s} \equiv W \quad (1\ \text{Watt})$$

Consider an amount of charge $\Delta q > 0$ that crosses a resistor R in time Δt.



The charge Δq losses pot./voltage ΔV_R when it crosses the resistor.

The corresponding loss of P.E. of the charge is

$$\Delta U = \Delta q \, \Delta V_R$$

If time to cross R is Δt, then the dissipated power is:

$$P = \frac{\Delta U}{\Delta t} = \frac{\Delta q \, \Delta V_R}{\Delta t} = \underbrace{\left(\frac{\Delta q}{\Delta t}\right)}_{I} \Delta V_R$$

∴ Power dissipated by a resistor is:

$$P = I \Delta V_R$$

know $\Delta V_R = \underline{IR}$ $\Rightarrow$ $P = I^2 R$

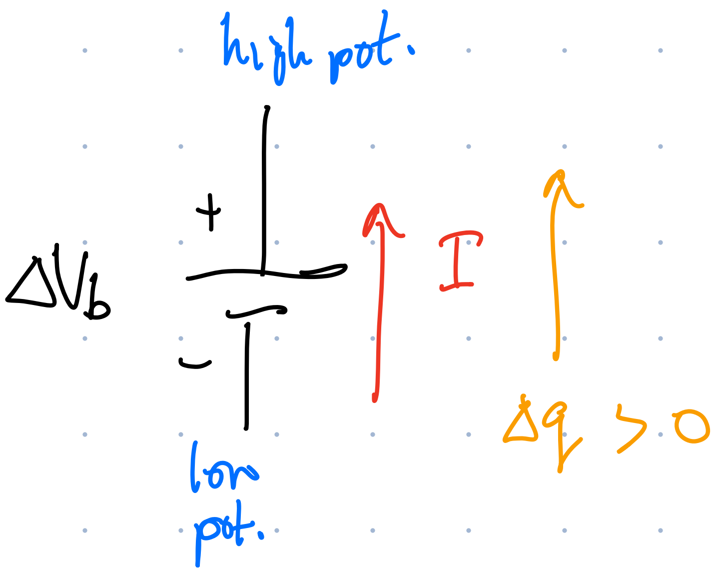
alternatively, $I = \dfrac{\Delta V_R}{\underline{R}}$ $\Rightarrow$ $P = \dfrac{(\Delta V_R)^2}{R}$

The power dissipated by a resistor is given by:

$$\boxed{P = I\,\Delta V_R = I^2 R = \frac{(\Delta V_R)^2}{R}}$$

The dissipated power is converted to heat/thermal energy.

In a similar way, we can determine the power <u>supplied</u> by a voltage source, such as a battery.



If $\Delta q > 0$ crosses battery from the neg. to pos. terminal in time Δt,

Change in energy is: $\Delta U = \Delta q\,\Delta V_b$

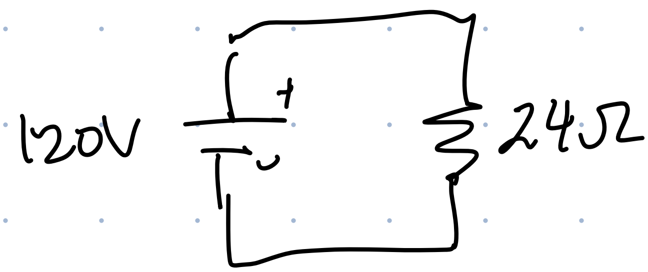
Power supplied is : $P = \frac{\Delta U}{\Delta t} = \left(\frac{\Delta q}{\Delta t}\right) \Delta V_b$

$$\boxed{P = I \Delta V_b}$$

In a circuit consisting of only batteries and resistors, the total power supplied by all batteries exactly balances the total power dissipated by all resistors.

Eg. The nichrome wire in a toaster has a resistance of $24\,\Omega$. If the outlet in the wall supplies 120V, find:

(a) current in the toaster.



$$I = \frac{120\,V}{24\,\Omega} = 5A.$$

(b) Power dissipated by resistor (nichrome wire)

$$P = I^2 R = (5A)^2 \, 24\Omega = 600W$$

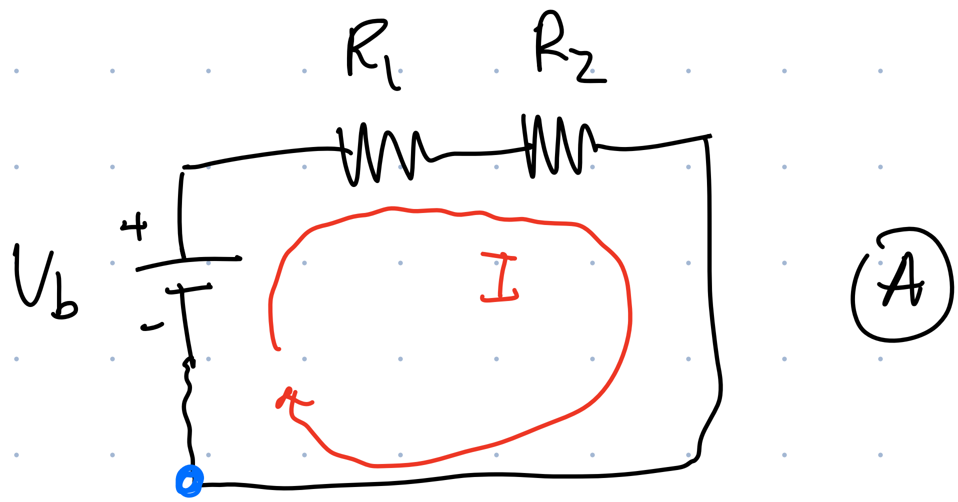
(c) Power supplied by outlet

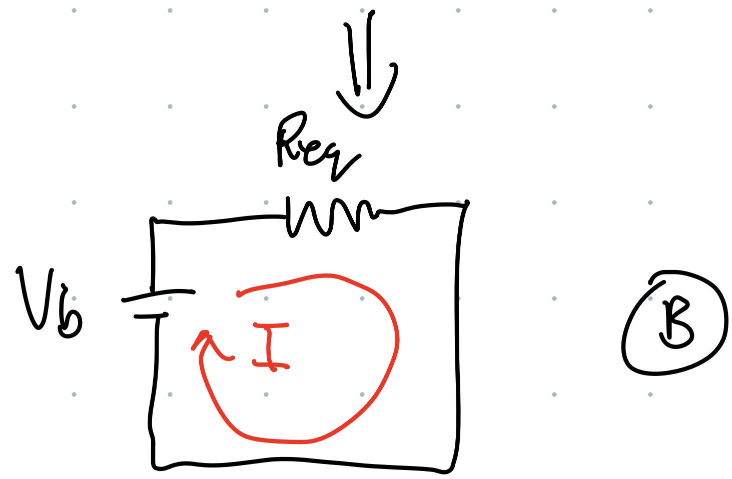
same. conservation of energy.

$$P = I \Delta V = (5A)(120V) = 600W$$

Combinations of Resistors (OSUPv2 Sec. 10.2)

Series





require current in both circuits to be the same when driven by the same battery

Do Kirchhoff loop rule for both circuits

$$\sum_{\text{closed loop}} \Delta V_i = 0$$

(A) $+V_b - IR_1 - IR_2 = 0$

$$\therefore V_b = I(R_1 + R_2) \quad \text{(i)}$$

(B) $+V_b - IR_{eq} = 0$

$$\therefore V_b = IR_{eq} \quad \text{(ii)}$$

(i) & (ii)

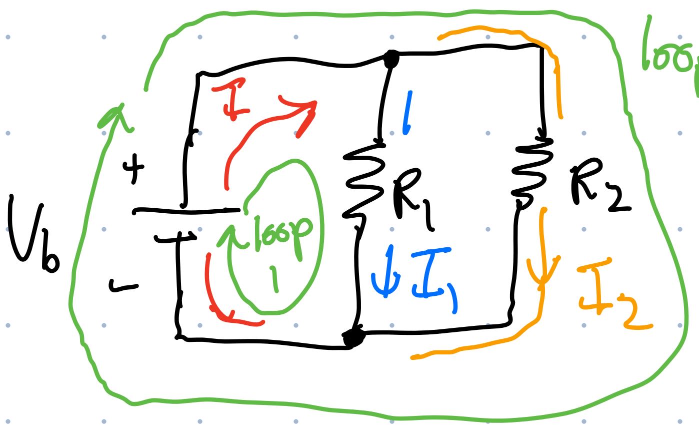
$$\Rightarrow \cancel{I}(R_1 + R_2) = \cancel{I} R_{eq}$$

$\therefore R_{eq} = R_1 + R_2$

In general, for N resistors in series

$$\boxed{R_{eq} = \sum_{i=1}^{N} R_i}$$

Resistors in Parallel



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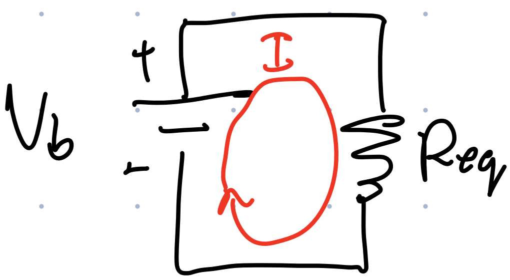

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Analysis of A

By the jcn rule (current in = current out)

$$I = I_1 + I_2$$

loop 1: Track changes in voltage.

$$+V_b - \Delta V_{R_1} = 0 \Rightarrow \Delta V_{R_1} = V_b$$

loop 2: $+V_b - \Delta V_{R_2} = 0 \Rightarrow \Delta V_{R_2} = V_b$

$\therefore$ Battery, R_1, R_2 all have the same voltage difference V_b.

$$I = I_1 + I_2$$

$$= \frac{\Delta V_{R_1}}{R_1} + \frac{\Delta V_{R_2}}{R_2} = \frac{V_b}{R_1} + \frac{V_b}{R_2}$$

$$\therefore I = V_b\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad i'$$

Analysis of B $\quad I = \dfrac{V_b}{R_{eq}}$ ii'

Require i' = ii'

$$\therefore \cancel{V_b}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\cancel{V_b}}{R_{eq}} \quad 1$$

$$\boxed{\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

In general, for N resistors in parallel,

$$\boxed{\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}}$$