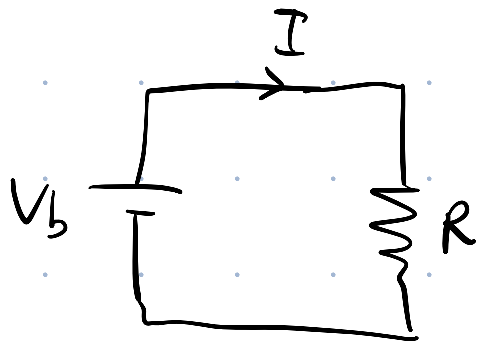
PHYS 121 March 5, 2025

- Complete Prairie Learn HW by Friday @ 23:59
- No Pre-Lab # 5

Recall:



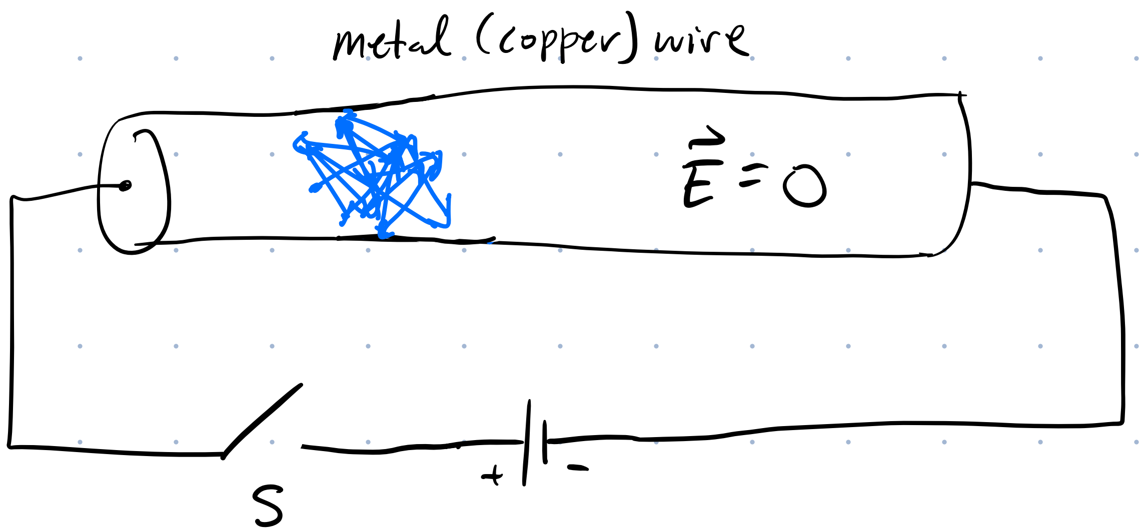
$$\boxed{V_R = IR}$$

voltage across resistor R

$$I = \frac{dQ}{dt}$$

Today: OSUPv2 Section 9.2

Model of Conduction in Metals

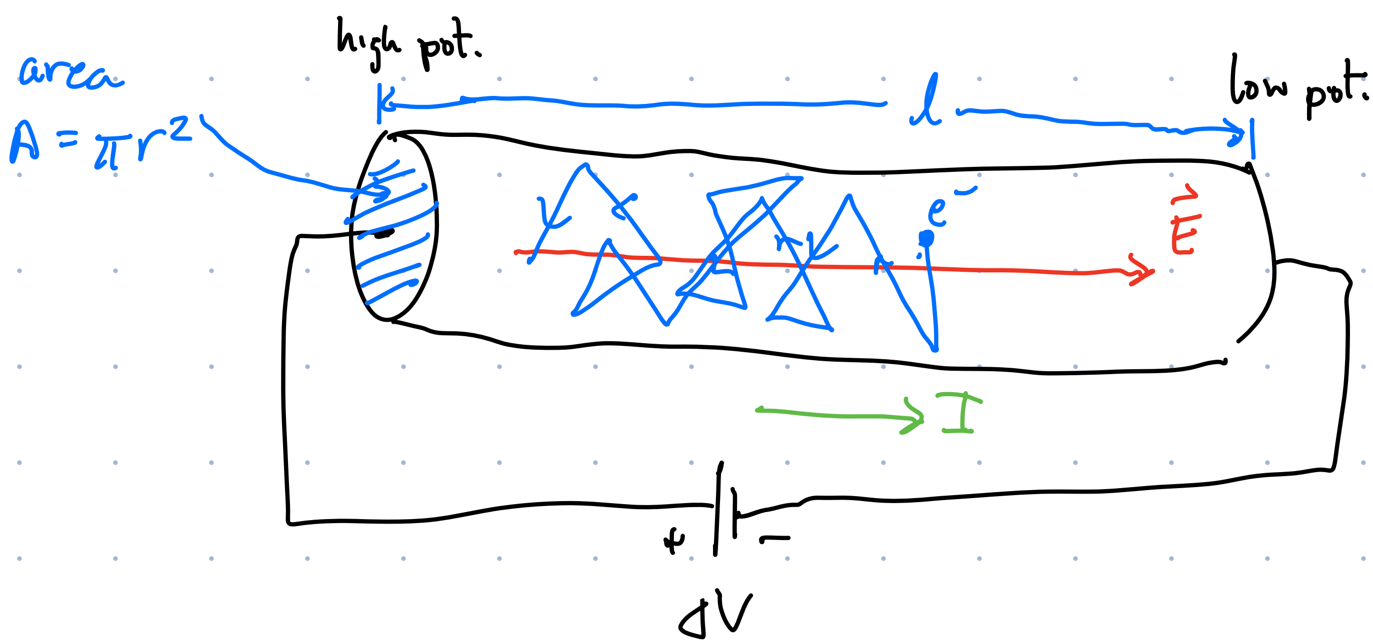


With swith open, there is no voltage across wire/resistor & $\vec{E} = 0$. $\Rightarrow$ conductor is in equil. No net flow of charge.

Conduction e^- constantly collide w/ atoms/impurities within the metal. After a collision the dir'n of of motion of e^- is radomized (Brownian motion).

In this case, there is no net motion of charge in any particular dir'n $\Rightarrow I = 0$.

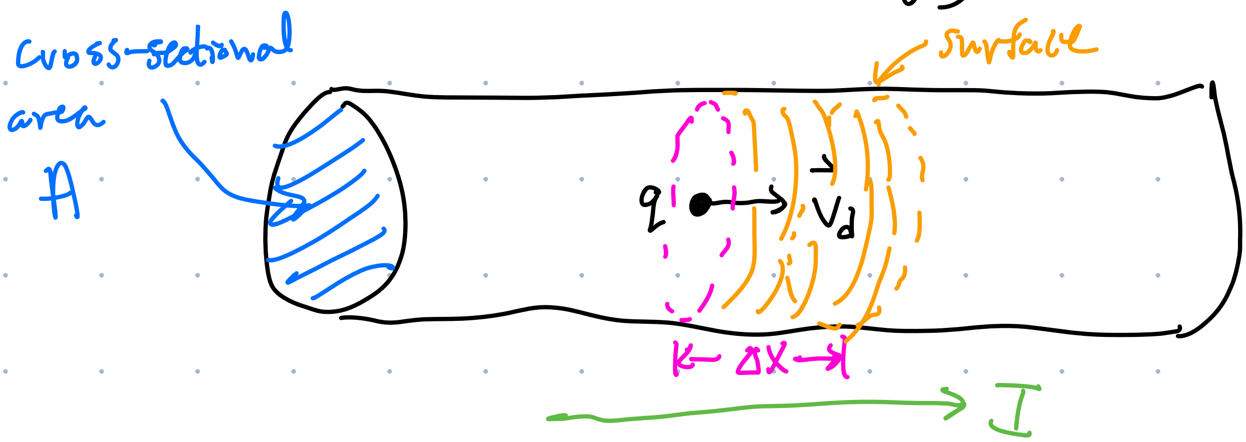
Now close the switch



Pot. diff. across wire, establishes an $\vec{E}$ in the wire which exerts a force on conduction e^-. This force cause a slow drift of e^- from right to left.

→ this charge flow establishes a current I.

Consider a conducting wire carrying current I



In time Δt, the total charge to cross surface is $\Delta Q = I \Delta t$ ①

Consider a pos. charge q moving w/ "drift velocity" v_d in the dir'n of I.

In time Δt, q moves a dist. $\Delta x = v_d \Delta t$

Any charge to the left of orange surface, but with Δx of it, will also cross the orange surface in time Δt.

The shaded volume is $A\Delta x$. If the conductor has an electron number density n:

$$n = \frac{\#\text{ of electrons}}{\text{volume}}$$

then the total no. of e^- in shaded region is

$$N_e = \underbrace{n}_{\frac{\text{no. } e^-}{\text{volume}}} \underbrace{A\Delta x}_{\text{shaded volume.}}$$

Since each e^- has charge e,

$$\underbrace{\Delta Q}_{\substack{\text{total charge to} \\ \text{cross surface in } \Delta t.}} = eN_e = enA\underbrace{\Delta x}_{V_d \Delta t}$$

$$\therefore \Delta Q = enAV_d \Delta t \quad (2)$$

Eq'ns (1) & (2) calc. the same quantity & must be equal:

$$(1) = (2)$$

$$I \cancel{\Delta t} = enAV_d \cancel{\Delta t}$$

$$\boxed{\therefore I = env_d A \quad \Rightarrow \quad V_d = \frac{I}{enA}}$$

Eg. For a copper wire of diameter $d = 1\,mm$ & $I = 1A$, what is V_d?

For copper, the electron number density is:

$$n = 8.3 \times 10^{28} \frac{e^-}{m^3}$$

$$V_d = \frac{I}{enA} = \frac{I}{en\left(\pi\left(\frac{d}{2}\right)^2\right)} \simeq 9.6 \times 10^{-5}\,m/s$$

$$\boxed{\simeq 0.1\,mm/s}$$

Return to $I = e n v_d A$

Note that I depends on the geometry of the wire (A). Sometimes it is convenient to define a "current density" $J = \frac{I}{A} = e n v_d$ which does not depend on geometry.

For some materials (metals), the current density J is prop. to the electric field E in the wire:

$$J \propto E$$

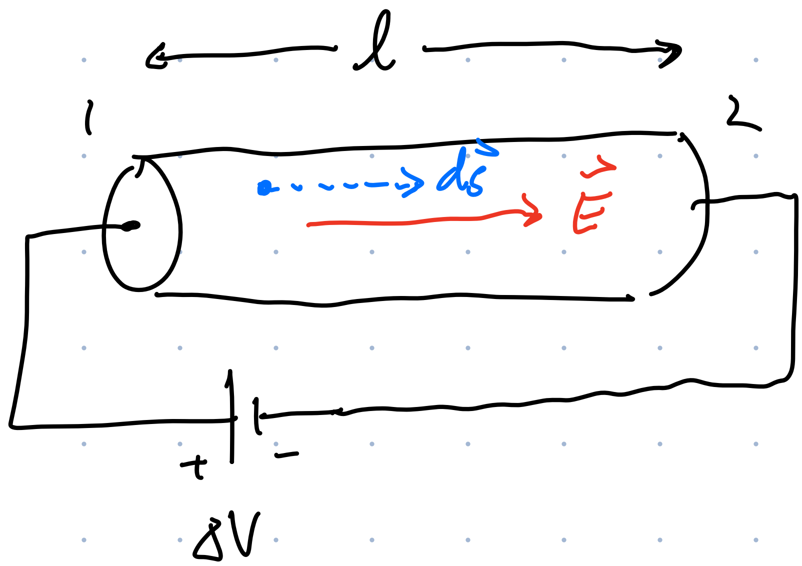
The constant of proportionality is called the conductivity & is given the symbol σ (sigma)

$$J = \sigma E \quad (3)$$

Materials for which Eq (3) holds are called "ohmic" materials & they obey "Ohm's Law."

Know $J = \frac{I}{A}$ (3a)

Consider the following conductor



Recall that pot. diff ΔV & $\vec{E}$ are related via:

$$\Delta V = -\int_1^2 \vec{E} \cdot d\vec{s}$$

If we assume that $\vec{E}$ is constant inside the conductor, then:

$$|\Delta V| = \int_1^2 E\,ds = E\int_1^2 ds = El$$

$$\therefore\ E = \frac{\Delta V}{l} \quad \text{(3b)}$$

(3) $J = \sigma E$

(3a) $J = \frac{I}{A}$ (3b) $E = \frac{\Delta V}{l}$

resistance of the wire.

$\therefore \frac{I}{A} = \sigma \frac{\Delta V}{l}$ solve for $\frac{\Delta V}{I} = R$

$$R = \frac{\Delta V}{I} = \frac{1}{\sigma}\frac{l}{A}$$

$$\boxed{\therefore R = \frac{1}{\sigma}\frac{l}{A} \qquad \underbrace{\Delta V = IR}_{\text{Ohm's Law}}}$$

$$[R] = \frac{V}{A} \equiv \Omega$$

ohm

$$1\text{ ohm} = \frac{1\text{ Volt}}{1\text{ amp}}$$

$$[\sigma] = \frac{1}{[R]}\frac{[l]}{[A]} = \frac{1}{\Omega}\frac{m}{m^2} = \frac{1}{\Omega \cdot m}$$

Often express resistance in terms of the resistivity ρ rather than the conductivity σ.

$$\rho = \frac{1}{\sigma}$$

$$\boxed{\therefore \quad R = \frac{1}{\sigma}\frac{\ell}{A} = \rho\frac{\ell}{A}}$$

conductivity

resistivity

Both material properties.