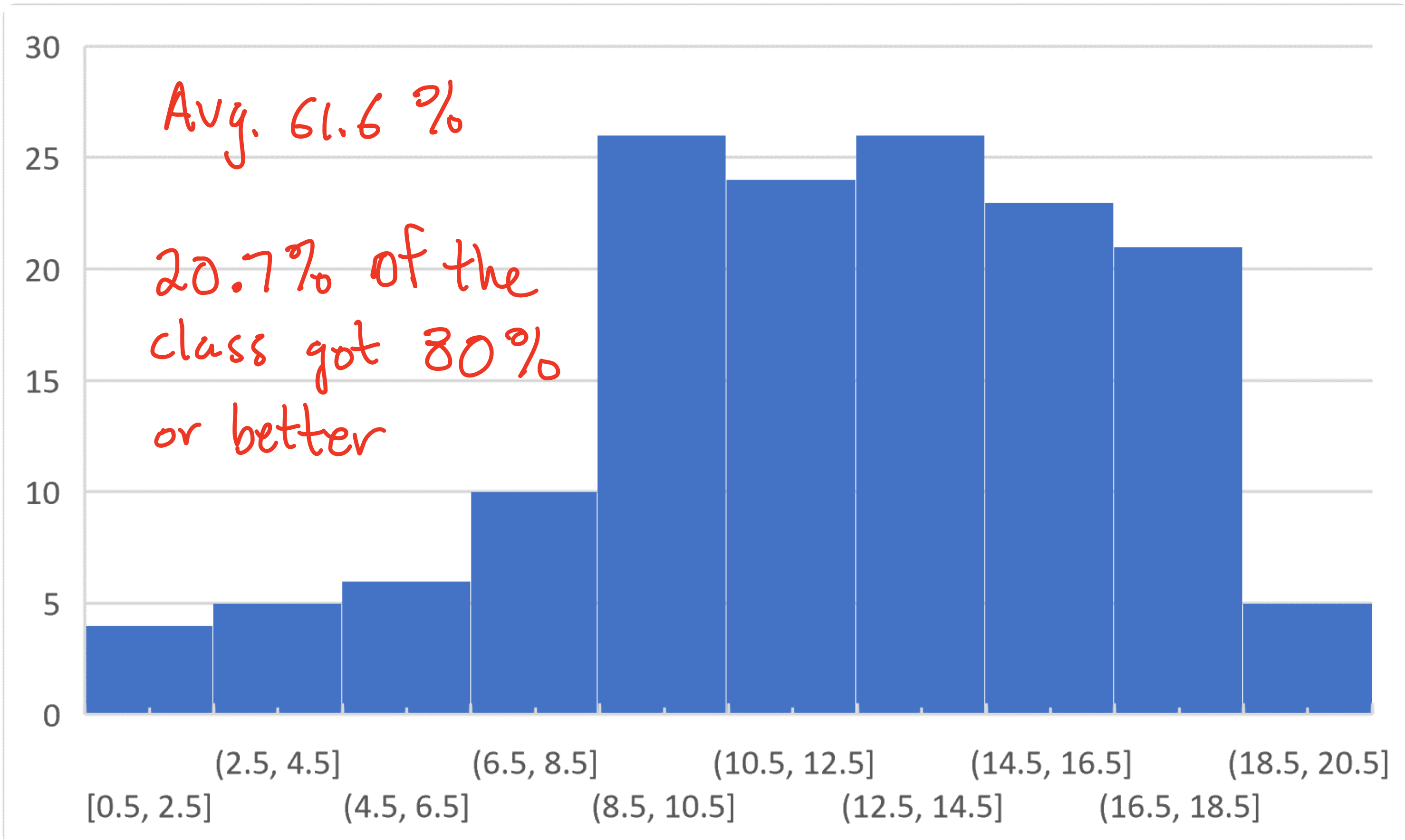
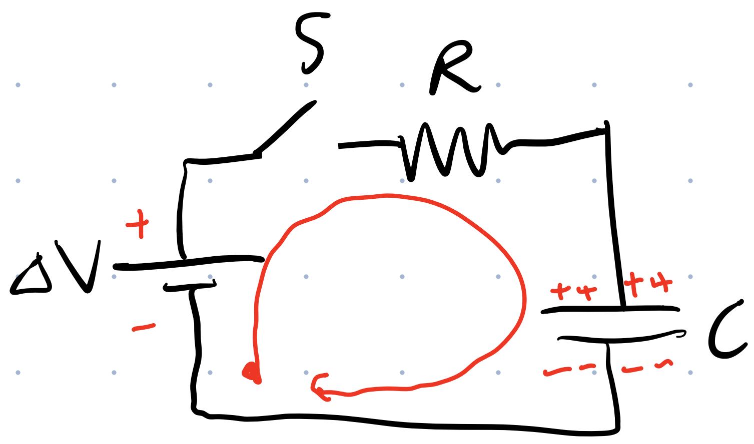
PHYS 121 March 2, 2025

- Complete PrairieLearn HW by Friday @ 23:59
- No Pre-Lab # 5
- Tutorials resume this week.

Midterm dist'n



Today: Charging a capacitor using an RC circuit.



$$C = \frac{Q}{\Delta V_C}$$

$$\Delta V_C = \frac{Q}{C}$$

Loop Rule: Change in voltage around a closed loop is zero.

battery   resistor   capacitor

$$\Delta V_{net} = 0 = \Delta V - \Delta V_R - \Delta V_C$$

$$0 = \Delta V - IR - \frac{Q}{C} \qquad (0)$$

Solve (0) for current I

$$I = \frac{\Delta V}{R} - \frac{Q}{RC}$$

define some constants: $I_0 = \frac{\Delta V}{R}$ $\Big\}$ current @ $t=0$ when cap is uncharged.

tau: $\tau = RC$ } time constant.

$\therefore$ Current at any time in our RC circuit is given by:

$$\boxed{I = I_0 - \frac{Q}{\tau}} \quad (1)$$

Since $I = \frac{\Delta Q}{\Delta t}$, we calc how much charge is added to cap. in time Δt using:

$$\boxed{\Delta Q = I \Delta t} \quad (2)$$

After finding ΔQ, know that the new charge on the cap is the original value of Q plus ΔQ.

$$Q \to Q + \Delta Q$$

Likewise:

$$t \to t + \Delta t$$

Strategy:

- Start w/ known values of ΔV, R, C $\Rightarrow$ const.

- Pick a value for the time interval Δt.

- Calc. $I_0 = \frac{\Delta V}{R}$ $\quad \tau = RC$

Start w/ $Q = 0$ @ $t = 0$

1. Use Eq. 1 to find I
2. Use Eq. 2 to find ΔQ
3. Update value of $Q \Rightarrow Q + \Delta Q$
4. Update value of $t \Rightarrow t + \Delta t$
5. Return to step 1 & iterate until Q reaches a stable value (equilibrium).

See course website for an example implementation.

For fun, let's see how to calc. Q & I exactly (won't test).

Start w/ Eq. ①

$$I = I_0 - \frac{Q}{\tau} \quad ①$$

$$I = \frac{dQ}{dt}$$ (instantaneous current)

$$\therefore \frac{dQ}{dt} = I_0 - \frac{Q}{\tau}$$

differential equation for the charge Q. Want to solve for $Q(t)$.

- Multiply by dt.

$$dQ = \left(I_0 - \frac{Q}{\tau}\right) dt$$

- Divide by $\left(I_0 - \frac{Q}{\tau}\right)$

$$\frac{dQ}{I_0 - \frac{Q}{\tau}} = dt$$

- Integrate both sides

$$\int_{Q=0}^{Q} \frac{dQ}{I_0 - \frac{Q}{\tau}} = \int_{t=0}^{t} dt$$

- make the substitution $u = I_0 - \frac{Q}{\tau}$

$$du = -\frac{dQ}{\tau} \Rightarrow dQ = \underline{-\tau\, du}$$

when $Q = 0$, $u = I_0$

$Q = Q$, $u = I_0 - \frac{Q}{\tau}$

$$\int_{u=I_0}^{I_0 - \frac{Q}{\tau}} \frac{-\tau\, du}{u} = t \Big|_0^t = t$$

I

$$\therefore\ -\tau \underbrace{\int_{u=I_0}^{I_0-Q/\tau} \frac{du}{u}}_{\ln u} = t$$

divide by $-\tau$

$$-\tau \ln u \Big|_{I_0}^{I_0-\frac{Q}{\tau}} = t$$

$$\ln u \Big|_{I_0}^{I_0-\frac{Q}{\tau}} = -\frac{t}{\tau}$$

$$\therefore \ln\left(I_0 - \frac{Q}{\tau}\right) - \ln I_0 = -\frac{t}{\tau}$$

use $\ln A - \ln B = \ln\left(\frac{A}{B}\right)$

$$\ln\left(\frac{I_0 - Q/\tau}{I_0}\right) = -\frac{t}{\tau}$$

Take exponential of both sides & solve for Q.

$$\frac{I_0 - \frac{Q}{\tau}}{I_0} = e^{-t/\tau}$$

$$\overset{1}{\frac{I_0}{I_0}} - \frac{Q}{I_0\tau} = e^{-t/\tau}$$

$$\frac{Q}{I_0\tau} = 1 - e^{-t/\tau}$$

$$\therefore Q = I_0\tau\left(1 - e^{-t/\tau}\right)$$

sub in $I_0 = \frac{\Delta V}{R}$, $\tau = RC$

$\therefore I_0\tau = C\Delta V$

$$\boxed{Q = C\Delta V\left(1 - e^{-t/\tau}\right)}$$

Charge on Cap vs t, when switch is closed @ $t=0$.

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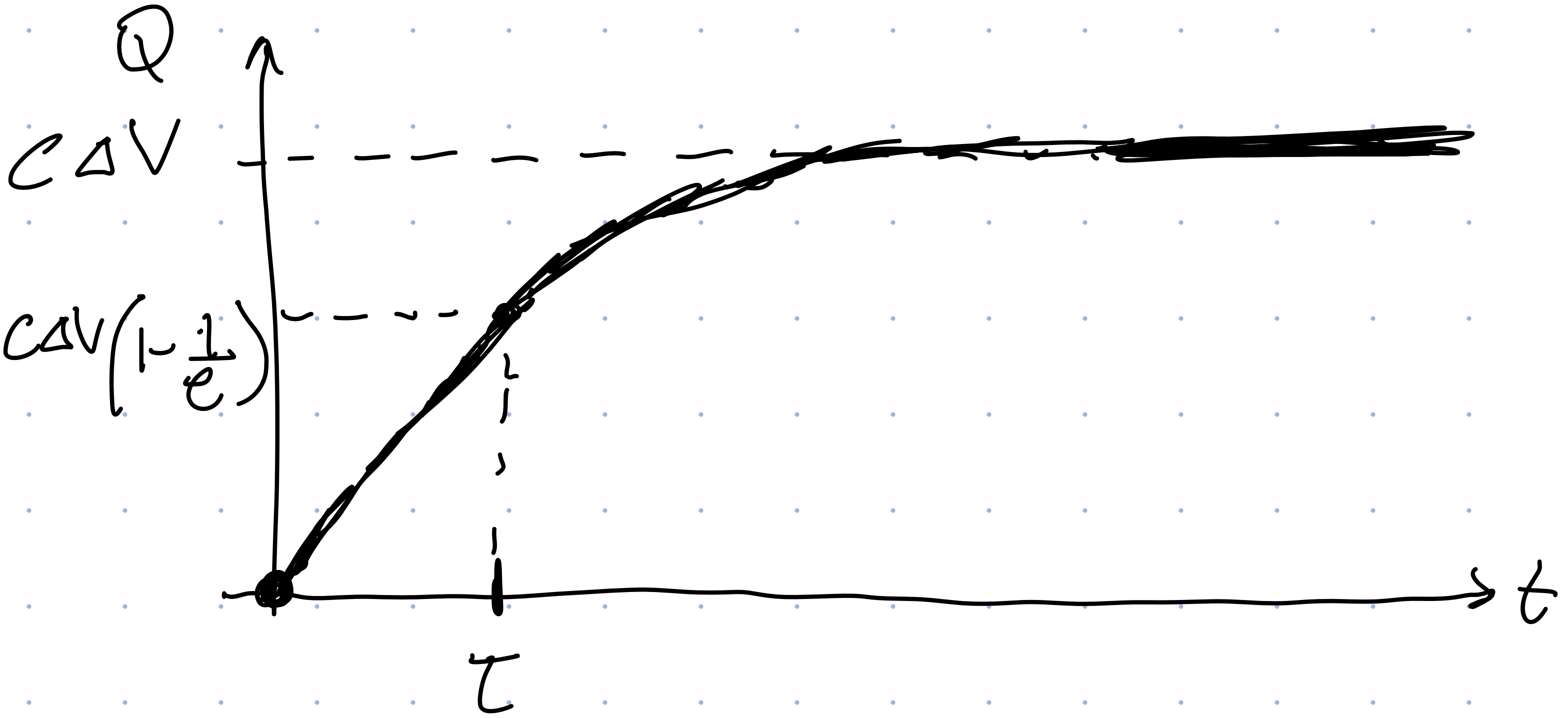

Can use Eq. ③ to calc. the current:

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[C\Delta V\left(1 - e^{-t/\tau}\right)\right]$$

$$= C\Delta V\left[\cancelto{0}{\frac{d}{dt}(1)} - \frac{d}{dt}\left(e^{-t/\tau}\right)\right]$$

$$= C\Delta V\left[-\left(-\frac{1}{\tau}\right)e^{-t/\tau}\right]$$

$$= \frac{C\Delta V}{\tau}e^{-t/\tau}$$ but $\tau = RC$

$$\boxed{\therefore I(t) = \frac{\Delta V}{R} e^{-t/\tau}}$$