PHYS 121 Feb. 28, 2025

<u>To do</u>:

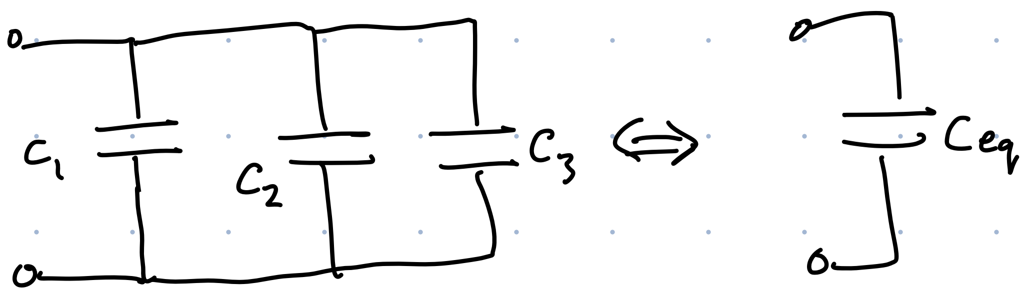
- Complete HW7 due $\boxed{\text{today}}$
- No Pre-Lab #5

<u>Last Time</u>:

- Definition of current

$$I = \frac{dQ}{dt} \qquad [I] = \frac{C}{s} = A$$

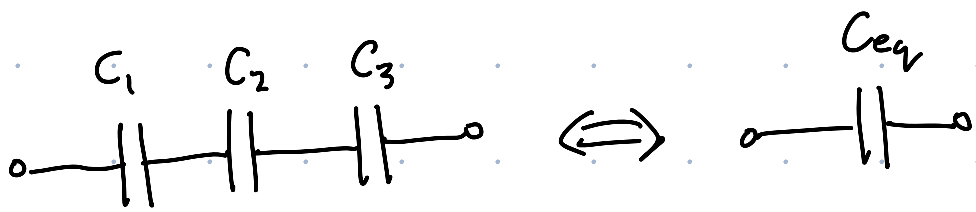
- Capacitors in Parallel



$$C_{eq} = C_1 + C_2 + C_3$$

For the parallel case, $C_{eq} > C_i \quad \forall \ i = 1..n$

Today: ▨ Capacitors in Series



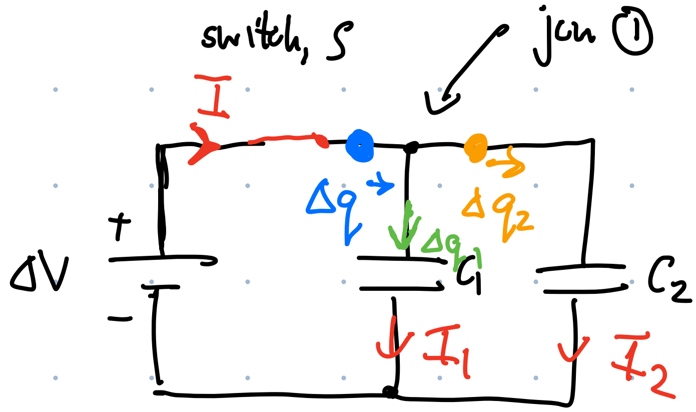
Will show that

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For the series case, $C_{eq} < C_i \quad \forall\ i = 1..n$

Along the way, we will also derive Kirchoff's laws for circuit analysis.

Imagine a simple parallel combination of two cap. & a battery w/ a switch:



When we close the switch, get a flow of charge or a current that transports pos. charge from the btm plates to the top plates.

Suppose we close the switch and watch the charge flow into & out of the jcn ① in the circuit.

In a time Δt, a packet of charge Δq approaches jcn ①.

At the jcn, some charge Δq_1 takes the path to C_1 & some takes the path to C_2.

Require $\Delta q = \Delta q_1 + \Delta q_2$ because charge is neither created nor destroyed at the jcn.

$\Rightarrow$ $\boxed{\text{Conservation of charge.}}$

If these observations are made in a time Δt, then

$$\frac{\Delta q}{\Delta t} = \frac{\Delta q_1}{\Delta t} + \frac{\Delta q_2}{\Delta t}$$

$\Downarrow$

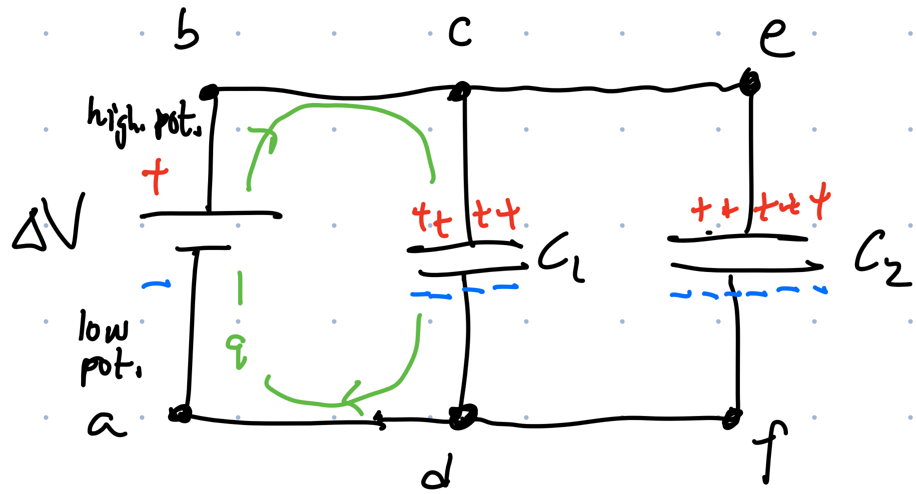
$$\boxed{I = I_1 + I_2 \qquad \text{Kirchhoff Jcn Rule}}$$

In general, Kirchhoff Jcn Rule:

current into a jcn = current out of a jcn

Second Kirchoff Rule:

Let's now imagine that our capacitors are fully charge.



Imagine taking a charge q around any closed loop in the circuit.

Consider the loop $a \to b \to c \to d \to a$

Keep track of changes in P.E.

Remiders: ① $C = \dfrac{Q}{|\Delta V|}$ $\quad \Delta V = \dfrac{Q}{C}$ ② $\Delta U = q\Delta V$

$a \to b$, gain volt. ΔV of battery,

gain P.E. $\Delta U = +q\Delta V$

$b \to c$, here $\Delta V = 0 \therefore \Delta U = 0$

$c \to d$, cross C_1 from pos. plate to neg. plate

loss voltage $\Delta V = -\frac{Q}{C_1}$, loss P.E. $\Delta U = -q\Delta V$

$d \to a$ $\Delta V = 0$, $\Delta U = 0$

$\therefore$ total change in P.E. around the loop is:

$$\Delta U_{net} = +q\Delta V + 0 - q\frac{Q}{C_1} + 0$$

$$= q\underbrace{\left(\Delta V - \frac{Q}{C_1}\right)}_{\text{zero}}$$

$\therefore \Delta U_{net} = 0$.

For any closed loop in a circuit, the change in P.E. of charge q is zero.

$$\Delta U_{neq} = q\,\Delta V_{net} = 0$$

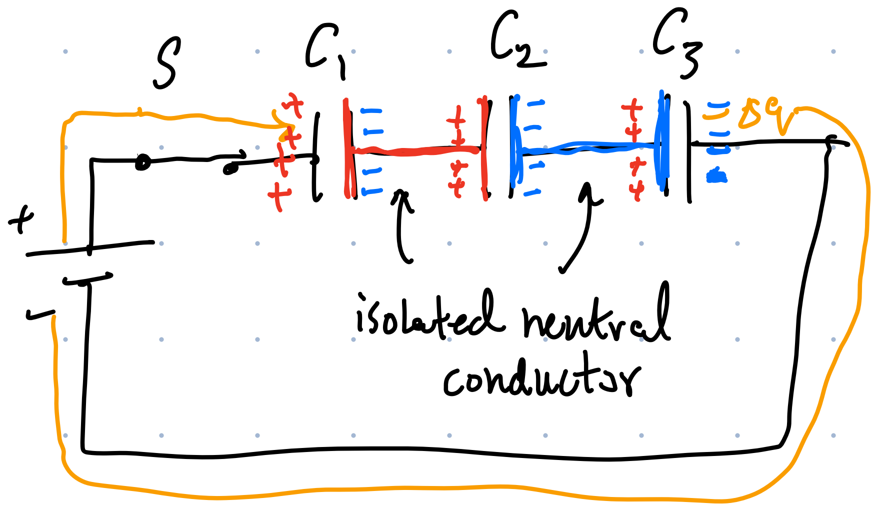
$$\Rightarrow \Delta V_{net}.$$

Kirchoff Loop Rule:

Net change in voltage around any closed loop in a circuit is zero.

$$\sum_i \Delta V_i$$

Capacitors in series



Start w/ switch open & capacitors uncharged.

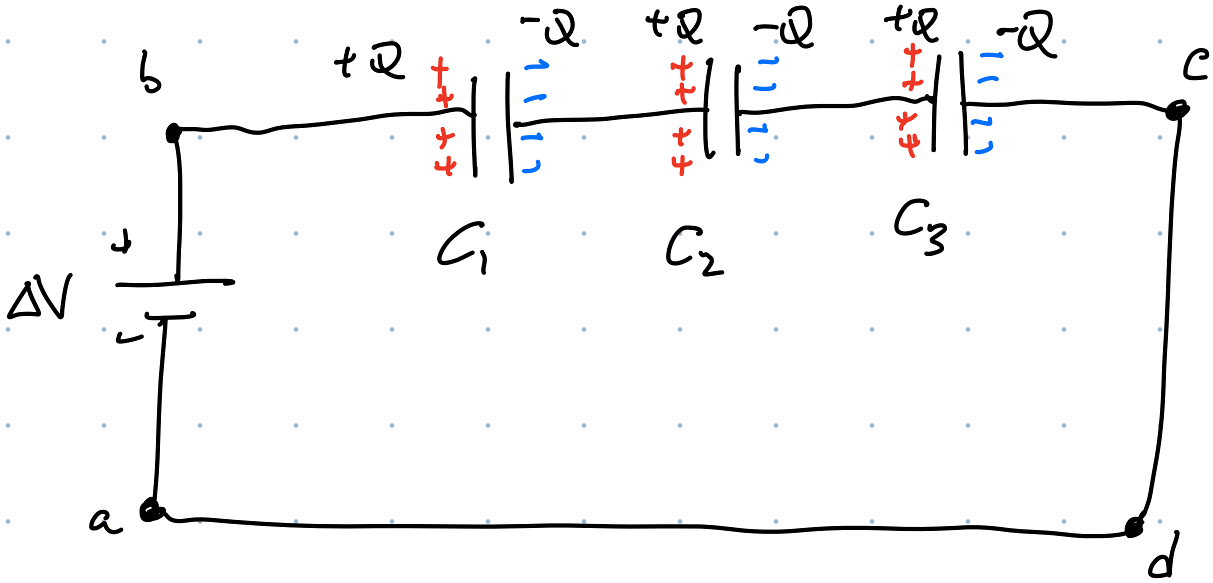
The red & blue conductors always remain neutral.
Charge cannot pass through the gaps of the capacitors.

When the switch is closed battery causes charge to transfer from right plate of C_3 to left plate of C_1.

- The neutral conductors (red & blue) become polarized (separation of ~~of~~ charge).

$\Rightarrow$ Each of the series capacitors carry the same charge

$$Q_1 = Q_2 = Q_3 \equiv Q$$



Apply the Kirchoff loop rule. $a \to b \to c \to d \to a$

$a \to b$: $+\Delta V$ (voltage of battery

$b \to c$: $-\Delta V_{C_1} - \Delta V_{C_2} - \Delta V_3$

$c \to d$: 0

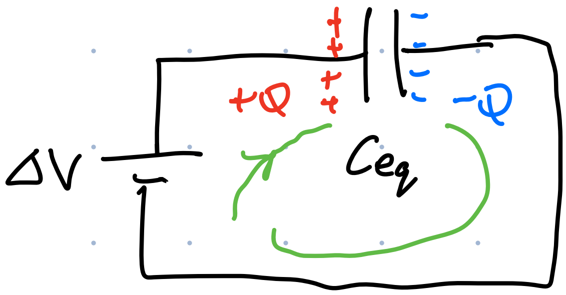
$d \to a$: 0

$$\Delta V_{net} = 0 = \Delta V - \Delta V_{C_1} - \Delta V_{C_2} - \Delta V_{C_3}$$

$$\therefore 0 = \Delta V - \frac{Q}{C_1} - \frac{Q}{C_2} - \frac{Q}{C_3}$$

$$\Delta V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \quad 1$$

Finally, want to represent the series combination as a single equiv. capacitor C_{eq}:



loop rule: $+\Delta V - \underbrace{\Delta V_{C_{eq}}}_{\frac{Q}{C_{eq}}} = 0$

$$\therefore \Delta V = \frac{Q}{C_{eq}} \quad (2)$$

Set (1) = (2)

$$\frac{\cancel{Q}^{1}}{C_1} + \frac{\cancel{Q}^{1}}{C_2} + \frac{\cancel{Q}^{1}}{C_3} = \frac{\cancel{Q}^{1}}{C_{eq}}$$

$$\Downarrow$$

$$\therefore \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

In general, for N caps. in series

$$\boxed{\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}}$$