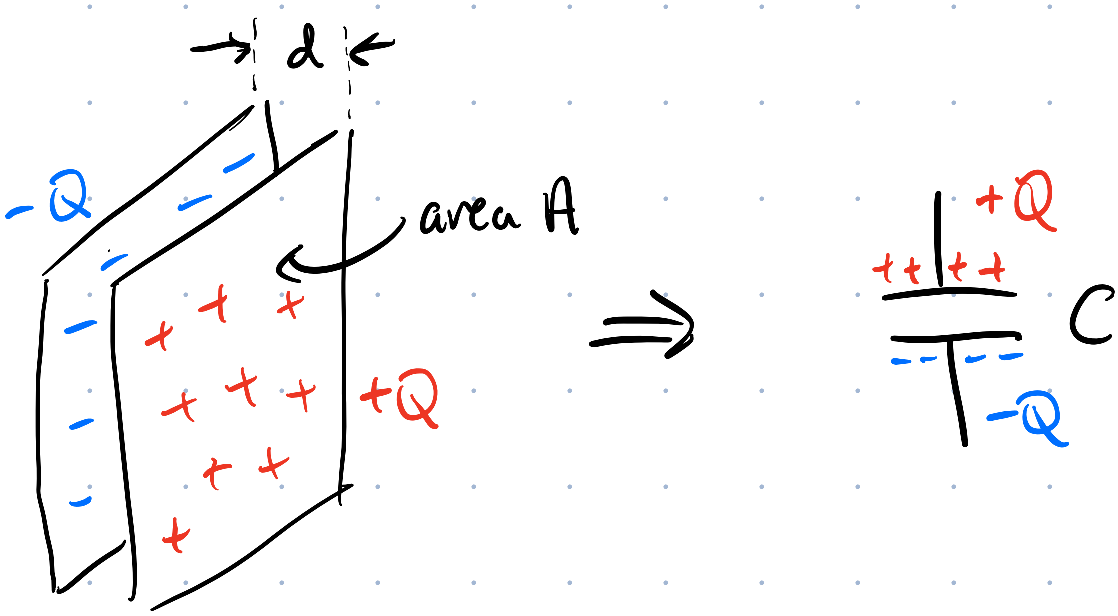
PHYS 121 Feb. 24, 2025

To do:

- Complete HW7 by 23:59 Feb. 28.
- Complete Pre-Lab #4 before start of Lab #4
- Midterm Feb. 26
  see course website for details
- No tutorials this week

Before the break: Parallel Plate capacitors

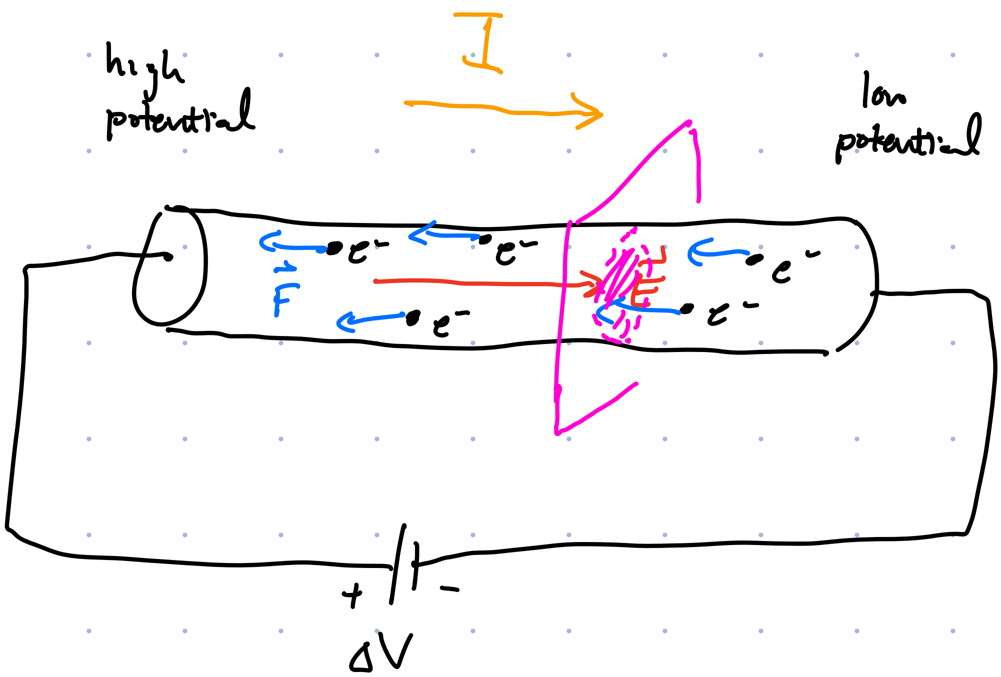


$$\boxed{C = \frac{Q}{|\Delta V|} = \varepsilon_0 \frac{A}{d}}$$

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} Q |\Delta V| = \frac{1}{2} C |\Delta V|^2}$$

Today: Start by discussing Lab #4.

Imagine connect a battery across a cylindrical conductor or a wire.



Recall that $\vec{E}$-fields point from high to low potential.

Since our conductor is not at a uniform potential & $\vec{E} \neq 0$ inside the conductor, this is not an equilibrium situation

$\Rightarrow$ Can & will have a net flow of charge or an electric current.

Potential differences drive flow of charge/current.

$\vec{E}$ in the conductor exerts forces on mobile e^- causing a flow of charge or a current.

Imagine keeping track of amount of charge Δq that crosses a surface intersecting the wire during time interval Δt.

Define average current as

$$I_{avg} = \frac{\Delta q}{\Delta t}$$

$$[I] = \frac{[\Delta q]}{[\Delta t]} = \frac{C}{s}$$

$$[I] = \frac{C}{s} = A$$

$$1 \text{ amp} = \frac{1 \text{ coulomb}}{1 \text{ sec.}}$$

If we shrink $\Delta t \to 0$, get the instantaneous current I.

$$\boxed{I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}}$$

In circuits, current is the dir'n of pos. charge flow.
For a metal (copper, aluminum) it is electrons (neg) that flow. ∴ current is in the opp. dir'n of the e^- flow.

Suppose we connect the same battery to different wires (diff. types of metal, diff. lengths, diff diameters)

Will find that for the same voltage, get different currents.

The resistance R of a wire is defined as

$$R = \frac{\Delta V}{I} \qquad [R] = \frac{V}{A} = \Omega$$

"ohm"

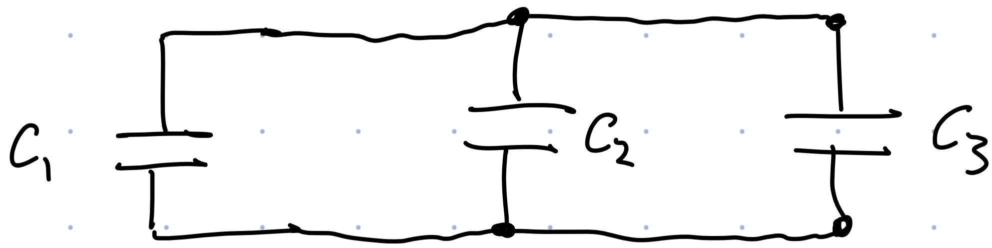
$$1\,\Omega = \frac{1\,V}{1\,A}.$$

In lab A4, you will investigate how the resistance of a wire depends on its diameter d.

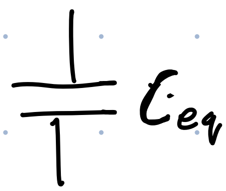
You will do the same investigation for hydraulic/water circuits. Compare & constrast the results.

Back to capacitors. → Combinations of capacitors.

Parallel Combinations



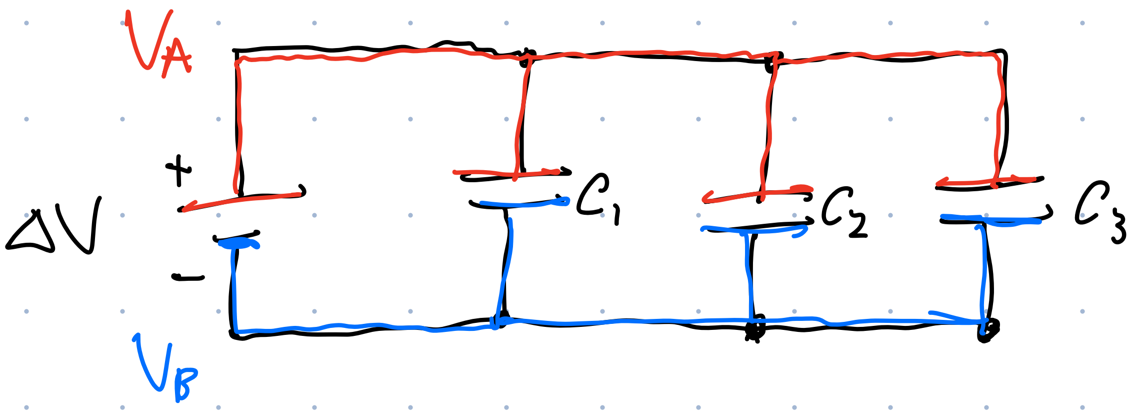
All top plates connected by a wire
" btm " " " " "
} Parallel combination.

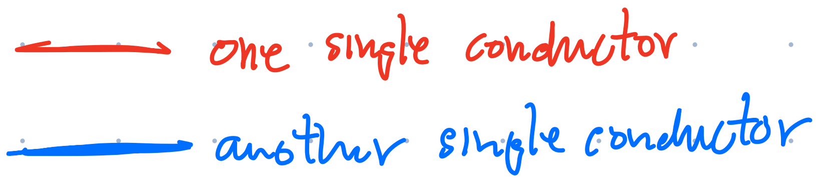


Want to represent the parallel comb. as a single "equivalent" capacitor C_{eq}.

$$\boxed{\text{Result: } C_{eq} = C_1 + C_2 + C_3}$$

Proof:





After the capacitors have charged, there is no more flow of charge ⇒ conductor are in equil.

⇒ blue & red conductors are each a diffent constant potential.

∴ the pot. diff. across each cap. is the same:

$$\Delta V_1 = \Delta V_2 = \Delta V_3 = V_A - V_B = \Delta V$$

pot. diff. provided by battery

Recall, by defintion, $C = \dfrac{Q}{\Delta V} \Rightarrow Q = C\Delta V$

$$Q_1 = C_1 \Delta V$$

$$Q_2 = C_2 \Delta V$$

$$Q_3 = C_3 \Delta V$$

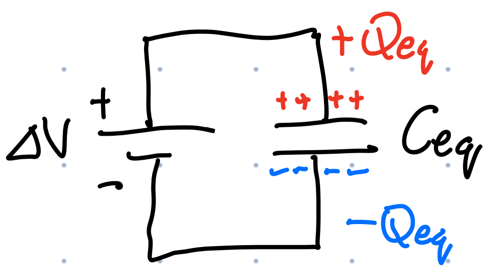
$\therefore$ the total charge Q stored by the parallel combination is:

$$Q = Q_1 + Q_2 + Q_3$$

$$= C_1 \Delta V + C_2 \Delta V + C_3 \Delta V$$

$$\boxed{\therefore Q = (C_1 + C_2 + C_3)\Delta V} \quad ①$$

Think about C_{eq}:



$$Q_{eq} = C_{eq} \Delta V$$

If C_{eq} is really equiv. to the parallel comb., we require $Q_{eq} = Q$.

$$\therefore \quad \boxed{Q = C_{eq} \Delta V} \quad (2)$$

$$(1) = (2)$$

$$(C_1 + C_2 + C_3) \cancel{\Delta V} = C_{eq} \cancel{\Delta V}$$

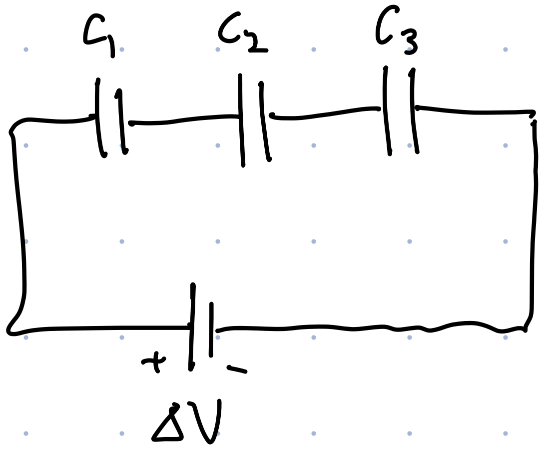
$$\boxed{\therefore \quad C_{eq} = C_1 + C_2 + C_3}$$

In general, for any no. of parallel cap., we have

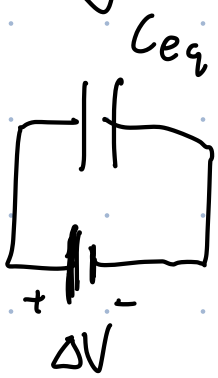
$$\boxed{C_{eq} = \sum_{i=1}^{n} C_i}$$

Note that $C_{eq} > C_i \quad \underbrace{\forall}_{\text{for all}} \quad i = 1..n$

For capacitors in series:



$\Downarrow$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

We will prove this result on Friday.

Note: Here, $C_{eq} < C_i \;\; \forall \; i = 1 .. n$