PHYS 121 Feb. 14, 2025

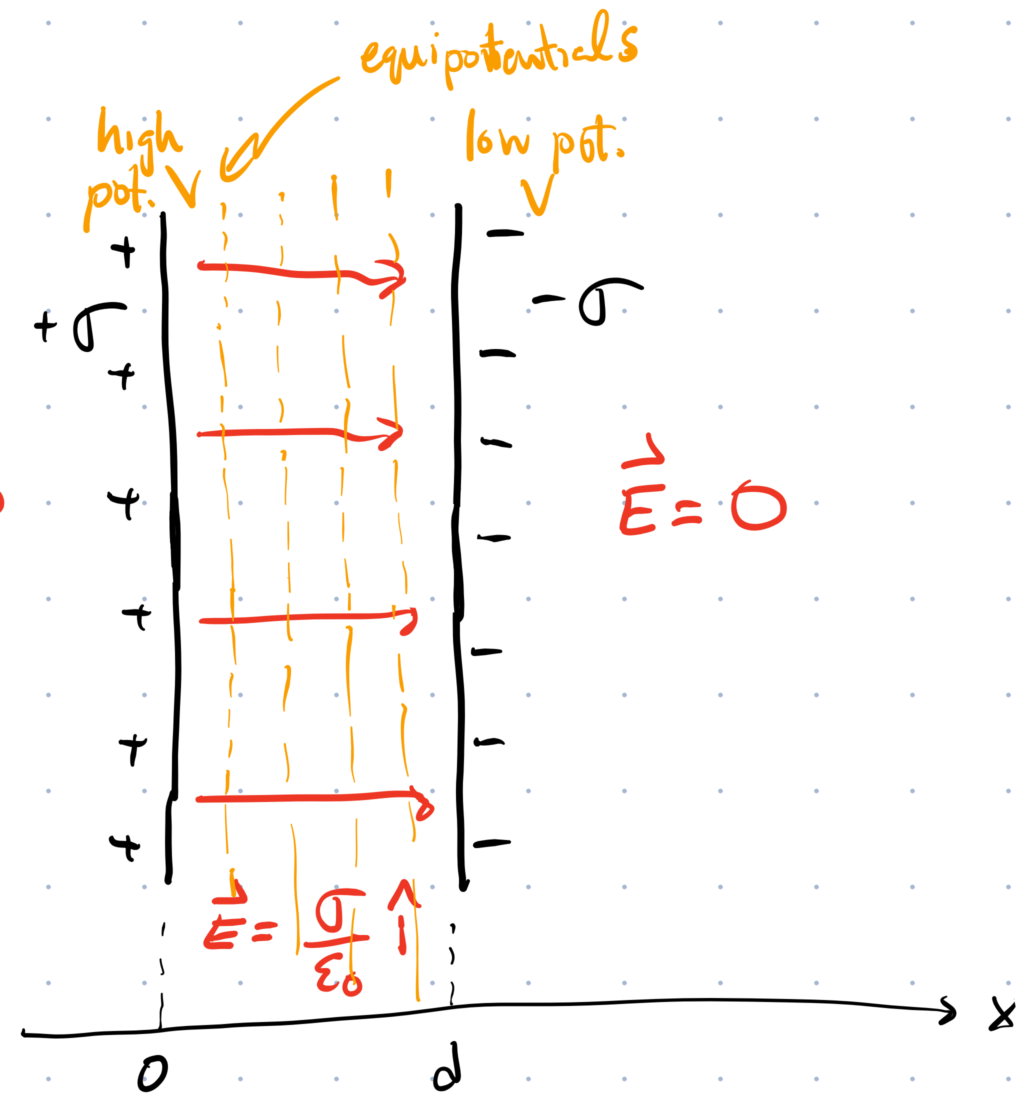
<u>To do:</u>

- Complete HW 6 by 23:59 Feb. 21.
- Complete HW 7 by 23:59 Feb. 28.
- Complete Pre-Lab #4 before start of Lab #4
- Midterm Feb. 26
  see course website for details
- No tutorials the week of the midterm

Last Time:

Parallel-Plate Capacitor

$\vec{E} = 0$

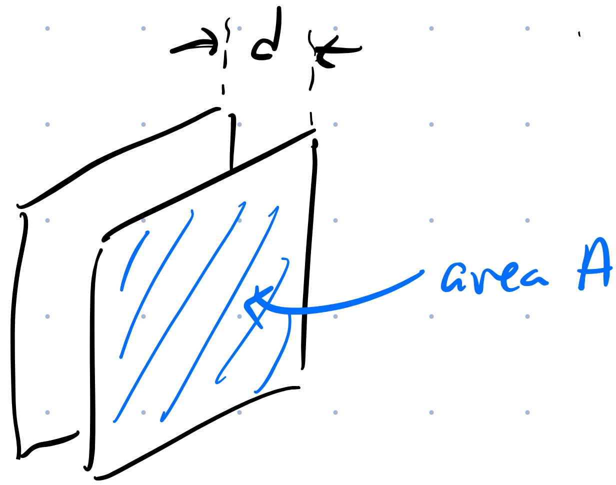


Each plate carries a charge per unit area:

$$|\sigma| = \frac{Q}{A}$$

where A is plate area

By definition:

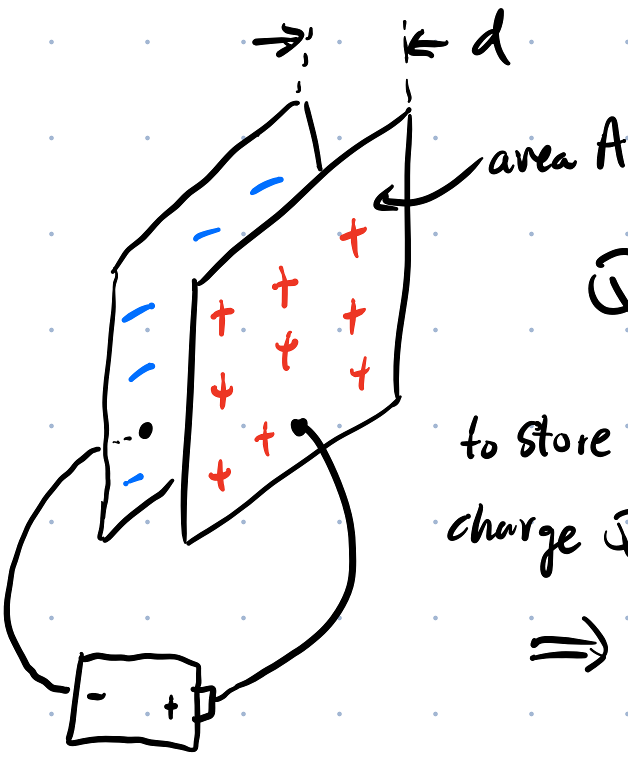


$$C = \frac{Q}{|\Delta V|}$$

$$|\Delta V| = Ed = \frac{\sigma}{\varepsilon_0} d = \frac{Qd}{A\varepsilon_0}$$

$$\boxed{\therefore C = \varepsilon_0 \frac{A}{d}}$$

Capacitance of a parallel-plate capacitor.



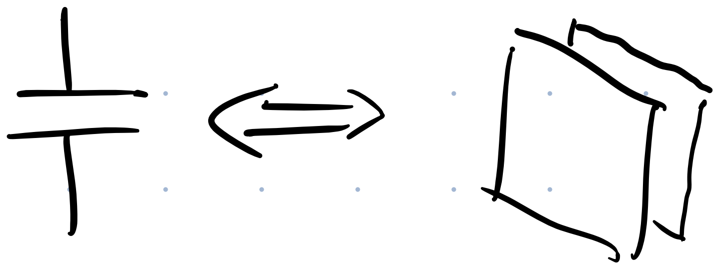
$$Q = C|\Delta V|$$

to store a large amount of charge Q, make C big.

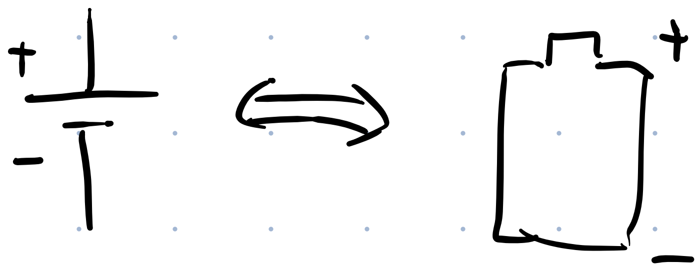
$\Longrightarrow$ large A, small separation d.

Circuit Symbols

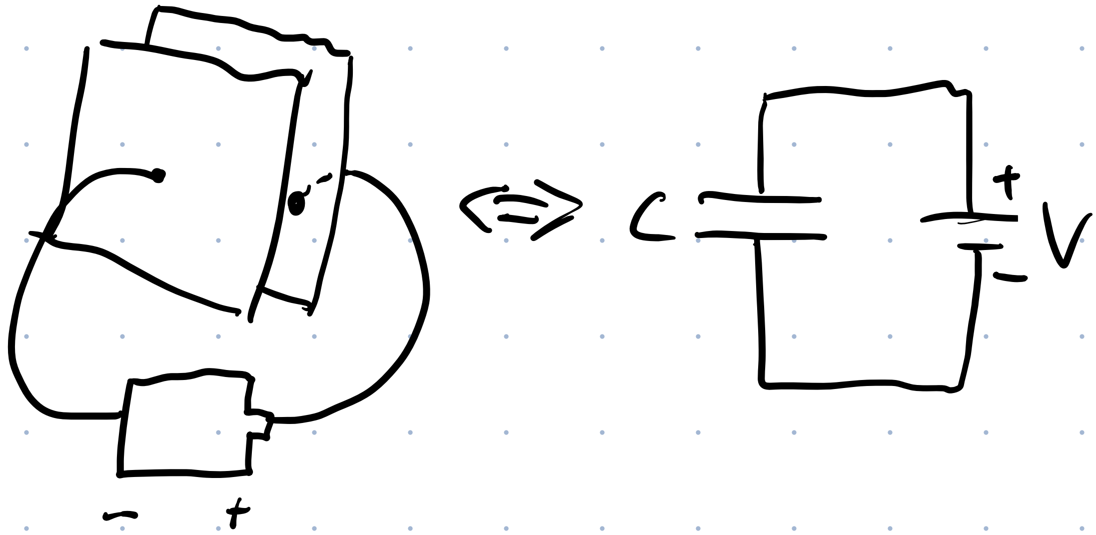
Schematic symbol for a capacitor



Schematic symbol for a battery is



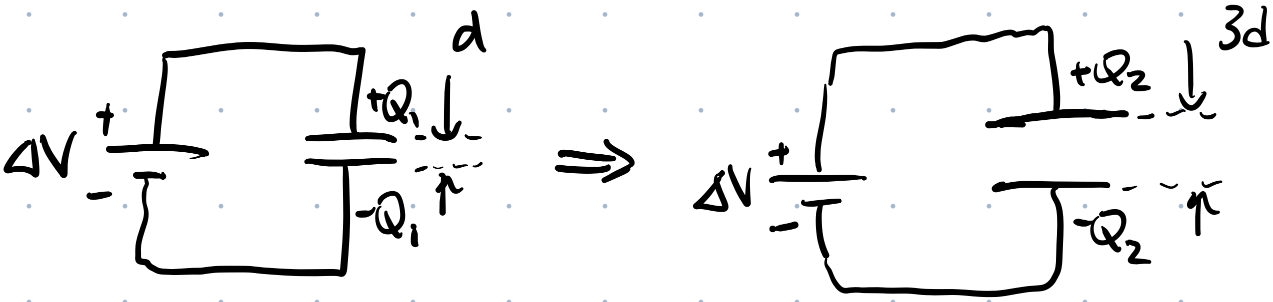
Schematic symbol for a resistor (next topic)



Eg. A parallel plate cap. is connected to a battery of voltage ΔV. The charge on the plates is $\pm Q_1$.

With the battery still connected, the capacitor plates are pulled apart s.t. $d \to 3d$.

What is the new charge on the plates Q_2?



Since the battery is always connected, ΔV is the same in both cases.

Know $Q = C|\Delta V|$ & $C = \varepsilon_0 \frac{A}{d}$

$$Q_1 = C_1 |\Delta V| = \varepsilon_0 \frac{A}{d} |\Delta V|$$

$$Q_2 = C_2 |\Delta V| = \varepsilon_0 \frac{A}{3d} |\Delta V|$$

$$= \frac{1}{3}\underbrace{\left(\varepsilon_0 \frac{A}{d} |\Delta V|\right)}_{Q_1}$$

$$Q_2 = \frac{1}{3} Q_1 \quad \checkmark$$

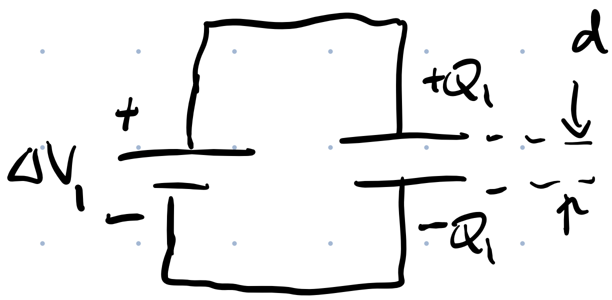
Eg. A parallel plate cap is connected to a battery of voltage ΔV. The charge on the plates is $\pm Q_1$.

After disconnecting the battery, the cap. plates are pulled apart s.t. $d \to 3d$.

(a) What is the new charge on the plates?

(b) What is the new voltage diff. between the plates?

Initial

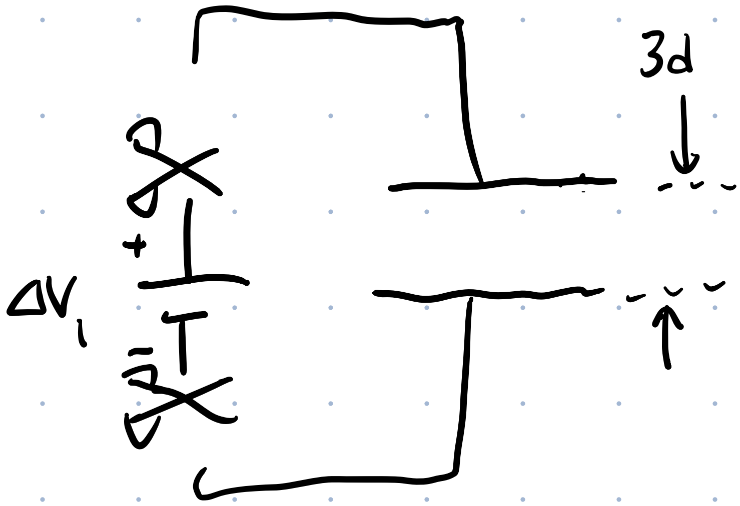


$$C = \frac{Q}{|\Delta V|}$$

$$Q_1 = C|\Delta V_1|$$

$$= \varepsilon_0 \frac{A}{d} |\Delta V_1|$$

Final



Since there is no connection from top plate to btm plate, there can be no transfer of charge to the charge.

$$\boxed{Q_1 = Q_2}$$

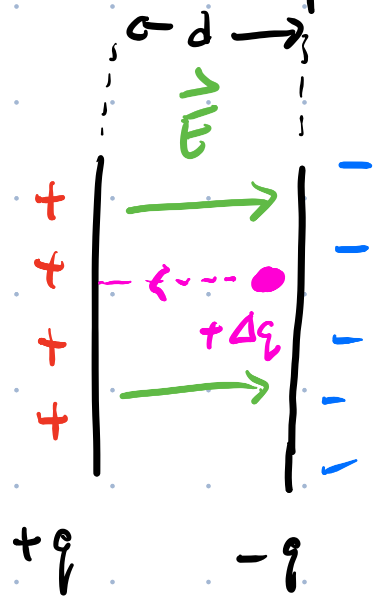
We need wires connecting the plates to something in order to change Q.

(b) $C = \frac{Q}{|\Delta V|} \Rightarrow |\Delta V| = \frac{Q}{C}$

initial	final
$Q_1 \equiv Q$	$Q_2 \equiv Q$
$C_1 = \varepsilon_0 \frac{A}{d}$	$C_2 = \varepsilon_0 \frac{A}{3d}$
$\lvert\Delta V_1\rvert = \frac{Q}{\varepsilon_0 \frac{A}{d}} = \frac{Qd}{\varepsilon_0 A}$	$\lvert\Delta V_2\rvert = \frac{Q}{\varepsilon_0 \frac{A}{3d}} = 3\underbrace{\left(\frac{Qd}{\varepsilon_0 A}\right)}_{\lvert\Delta V_1\rvert}$
	$\therefore \lvert\Delta V_2\rvert = 3\lvert\Delta V_1\rvert$

Energy Stored by a charge Capacitor

Start w/ a capacitor charged to $\pm q$



Know $E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$

Also know $\Delta V = Ed = \frac{qd}{\varepsilon_0 A}$

Consider moving a small pos. charge Δq from neg. plate to positive plate. Move it through the electric field E between the plates.

The energy cost associated w/ moving this charge is

$$\Delta U = \Delta q \, \Delta V$$

$$\therefore \Delta U = \left(\frac{d}{\varepsilon_0 A}\right) q \Delta q$$

The total energy required to charge the capacitor is the sum of all ΔU starting from an uncharge cap ($q=0$) & ending w/ a fully-charge cap. ($q=Q$)

$$U = \sum \Delta U = \sum_{q_i=0}^{q_f=Q} \left(\frac{d}{\varepsilon_0 A}\right) q \Delta q$$

In the limit that $\Delta q \to 0$

$$U = \int_{q_i=0}^{q_f=Q} \left(\frac{d}{\varepsilon_0 A}\right) q \, dq$$

$$= \frac{d}{\varepsilon_0 A} \int_{q_i=0}^{q_f=Q} q \, dq = \frac{d}{\varepsilon_0 A} \left(\frac{1}{2} q^2\right) \Bigg|_{q_i=0}^{q_f=Q}$$

$$\therefore U = \frac{1}{2} \frac{d Q^2}{\varepsilon_0 A}$$

$\frac{1}{C}$

Recall that $C = \frac{\varepsilon_0 A}{d}$

$$\boxed{\therefore\ U = \frac{Q^2}{2C}}$$ Energy stored by a charged capacitor.

Substitute $C = \frac{Q}{|\Delta V|}$

$$U = \frac{Q^{\cancel{2}}}{2\left(\frac{\cancel{Q}}{|\Delta V|}\right)} = \boxed{\frac{1}{2} Q |\Delta V|}$$

Let's now express $Q = \underline{C|\Delta V|}$

$$\therefore\ U = \frac{1}{2}\left(C|\Delta V|\right)|\Delta V| = \boxed{\frac{1}{2} C |\Delta V|^2}$$

$\therefore$ There are 3 equivalent ways of expressing the energy of a charge capacitor:

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} Q |\Delta V| = \frac{1}{2} C |\Delta V|^2}$$