PHYS 121 Feb. 10, 2025

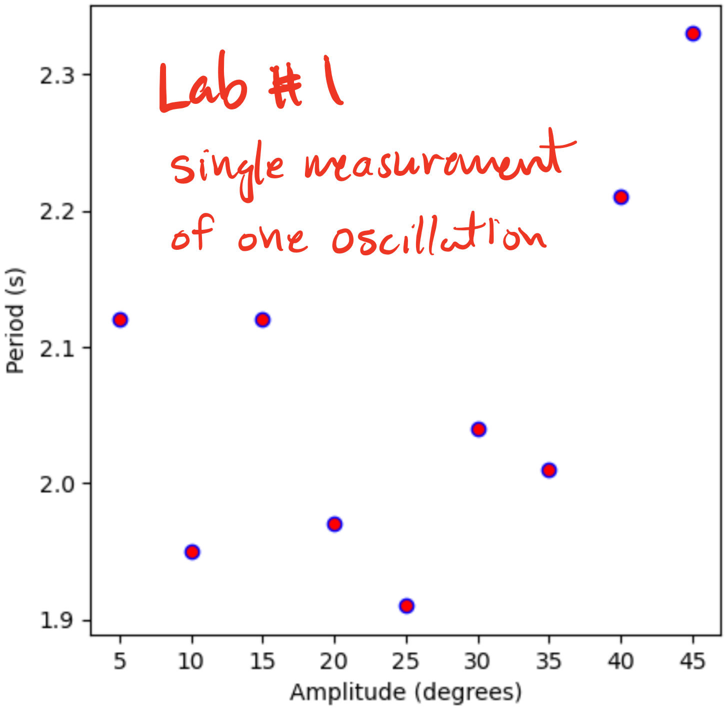
<u>To do</u>: - Complete HW6 by 23:59 Feb. 21.

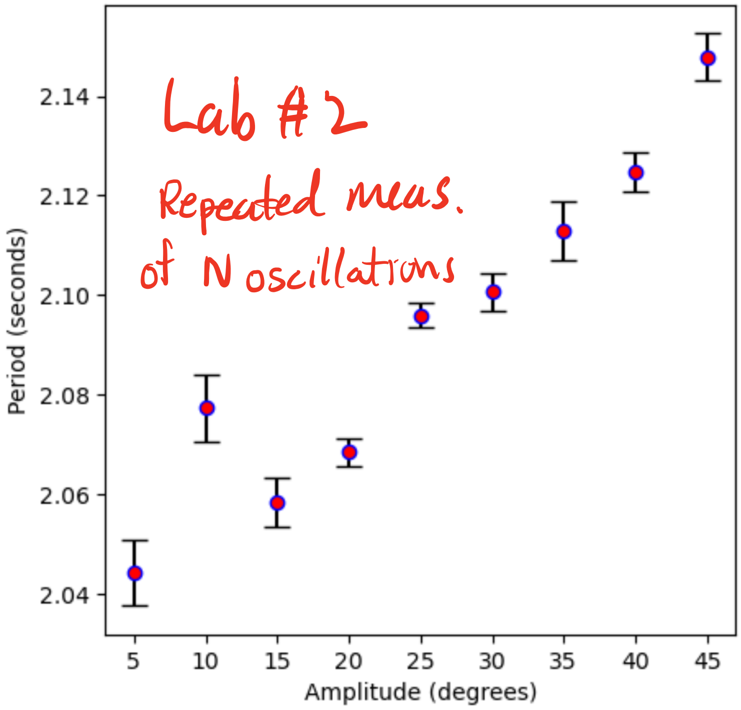
- Complete Pre-Lab #3 before start of Lab #3

- If participating in Hands-On Bonus project, please email me your project proposal by 23:59 $\boxed{\text{today}}$

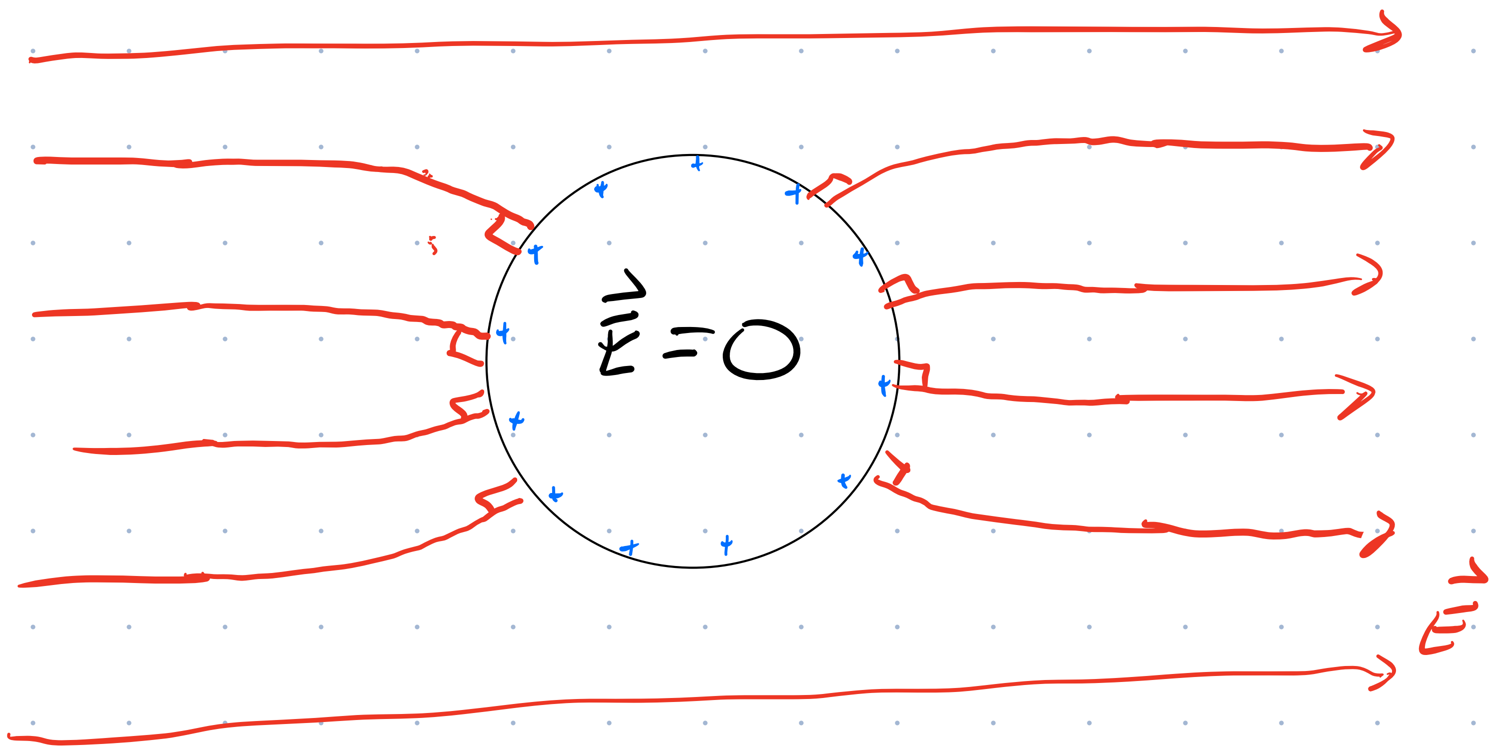
- Midterm Feb. 26
  see course website for details

Pendulum period vs amplitude





Last Time: Conductors in Equilibrium



① $\vec{E} = 0$ inside conductors

② $\vec{E}$ just outside a conductor is $\perp$ to conductor's surface

③ All excess charge resides on conductor surfaces.

Two classes ago:

- Potential energy of a pair of pt. charges:

$$U = \frac{k_e q Q}{r}$$

- Electric Potential (Voltage):

$$V = \frac{U}{q}$$

- For a pt. charge Q

$$V = \frac{k_e Q}{r}$$

- Finding changes in potential from $\vec{E}$

$$\Delta V = -\int_1^2 \vec{E} \cdot d\vec{s}$$

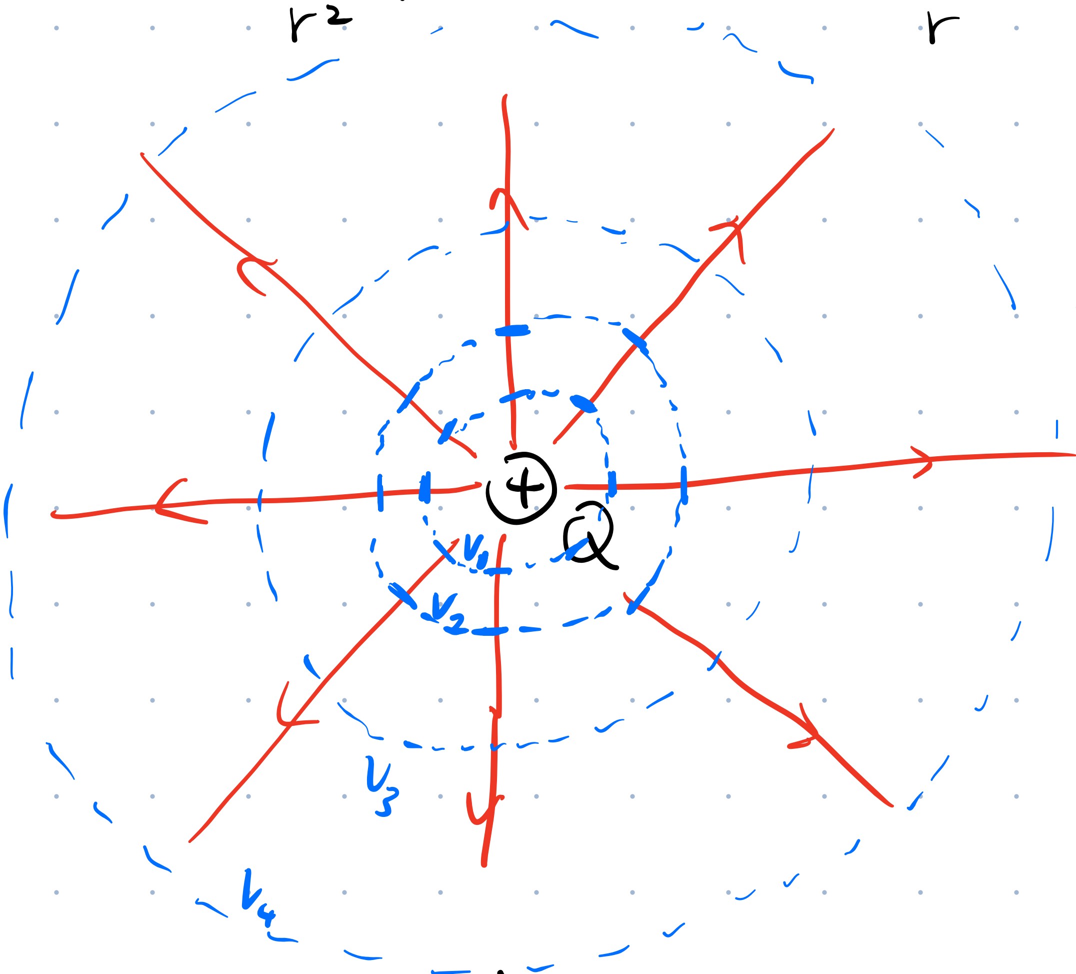
- Finding components of $\vec{E}$ from V

$$E_s = -\frac{dV}{ds} \qquad s = x, y, z, r$$

Return to a point charge Q

$$\vec{E} = \frac{k_e Q}{r^2}\,\hat{r} \qquad V = \frac{k_e Q}{r}$$

maintain
$V_4 - V_3$
$= V_3 - V_2$
$= V_2 - V_1$
$= \Delta V$



— $\vec{E}$-field
---- equipotentials

To this picture of $\vec{E}$, want to add lines of constant potential V. (Equipotential lines).

Start $\quad \Delta V = -\int \vec{E}\cdot d\vec{s}$

If we move $\perp$ to $\vec{E}$, then $\vec{E}\cdot d\vec{s} = E\,ds\,\cancel{\cos 90^\circ}^{\,0}$
$= 0$

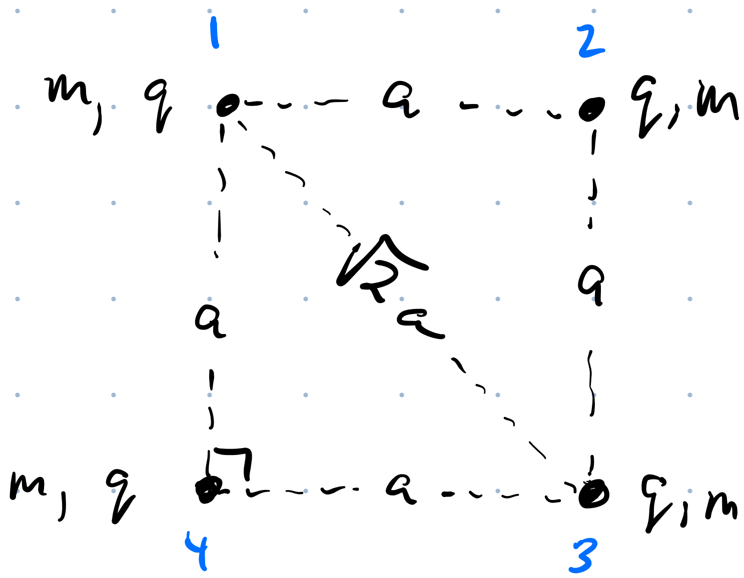
$\therefore \Delta V = 0 \Rightarrow \Delta V = V_2 - V_1 \Rightarrow V_2 = V_1$

If we move along a path that is always $\perp$ to $\vec{E}$, then the potential V remains const. $\Rightarrow$ equipotential line.

For a pt. charge, the equipotential lines are circles that surround Q.

If we want ΔV to be the same between each equipotential, need $d\vec{s}$ steps to increase as we move away from Q and $\vec{E}$ decreases.

Potential Energy Example



Four identical pt. charges w/ charge q & mass m are at rest at corners of a square of sides length a. At $t=0$, the particles are released. How fast are they moving when they are infinitely far from one another?

Use conservation of energy $E = K + U = \text{const.}$

(a) Find initial P.E. of the system.

For a pair of pt. charges $U = \frac{k_e q Q}{r}$

In our example, we have 6 different pairs of charges U_{12}, U_{13}, U_{14}, U_{23}, U_{24}, U_{34}

In our example:

$$U_{12} = U_{23} = U_{34} = U_{14} = \frac{k_e q^2}{a}$$

$$U_{13} = U_{24} = \frac{k_e q^2}{\sqrt{2}a}$$

$$U_i = 4U_{12} + 2U_{13}$$

$$= \frac{4k_e q^2}{a} + \frac{2k_e q^2}{\sqrt{2}a}$$

$$\boxed{U_i = \frac{k_e q^2}{a}\left[4+\sqrt{2}\right]}$$ initial P.E.

Initially, the charges are at rest. $\therefore K_i = 0$

$$\therefore E_i = K_i + U_i = \frac{k_e q^2}{a}(4+\sqrt{2})$$

(b) Find the mech. energy after the charges are infinitely far apart.

$$K_f = 4\left(\tfrac{1}{2}mv^2\right) = 2mv^2$$

4 identical particles

$U_f = ?$

Still have 6 pairs of charges s.t.

$$U = \frac{k_e q^2}{r}$$

but $r \to \infty$ so $U \to 0$

$$U_f = 0$$

$$E_f = K_f + U_f = 2mv^2$$

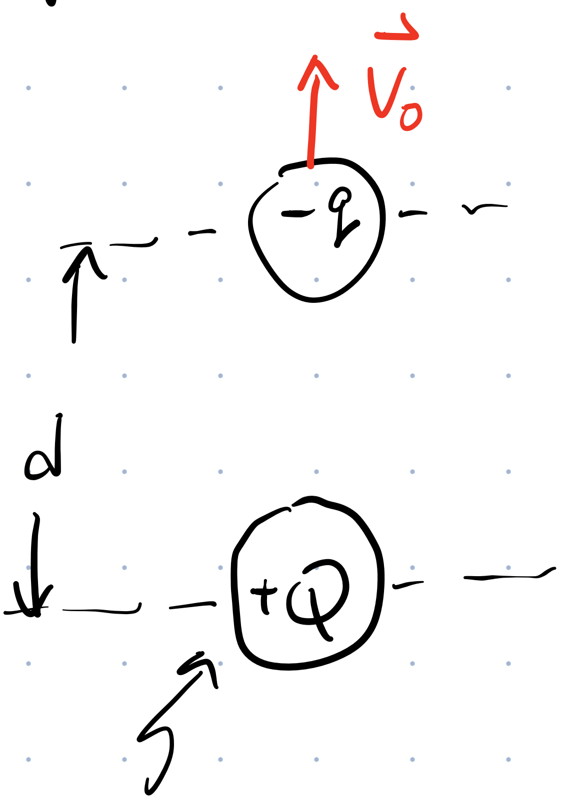
(c) Set $E_i = E_f$ & solve for v.

$$\therefore \frac{k_e q^2}{a}\left(4 + \sqrt{2}\right) = 2mv^2$$

$$\boxed{\therefore v = \sqrt{\frac{k_e q^2}{ma}\left(2 + \frac{1}{\sqrt{2}}\right)}}$$

initial dist. between charges.

Eg. Escape Velocity.



held fixed in place the entire time.

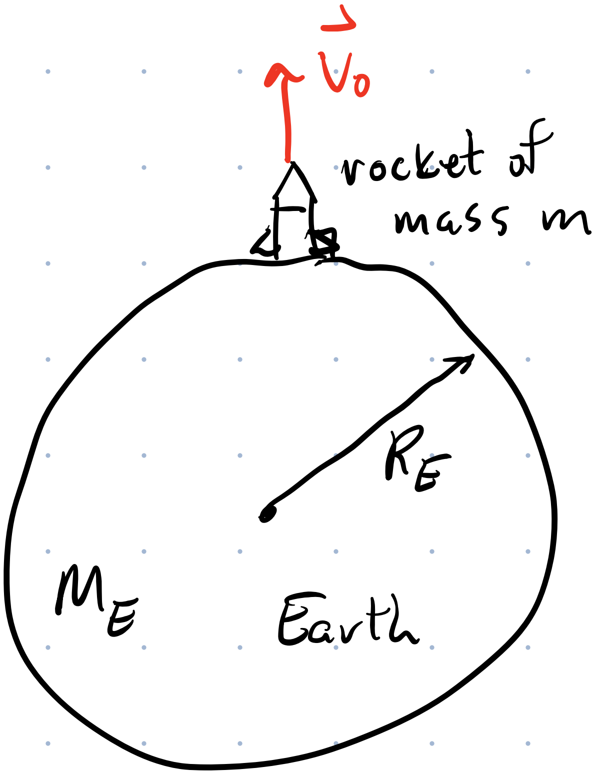
How fast must the initial speed V_0 be s.t. $-q$ & $+Q$ never come together.

(What is the speed require to escape the pull of $+Q$)?

b/c of neg. charge

$$U = \frac{-k_e q Q}{r}$$

Parallel Prob.:



What is the minimum value of v_0 s.t. rocket never returns to Earth?

$$U_G = -\frac{G M_E m}{r}$$

charges	rocket
initial $U_i = -\frac{k_e q Q}{d}$	$U_i = \frac{-G M_E m}{R_E}$
$K_i = \frac{1}{2} m v_0^2$	$K_i = \frac{1}{2} m v_0^2$

To ensure that charge/rocket never returns, require the speed equal zero only when $r \to \infty$.

Final

$U_f = 0 \quad (r \to \infty)$

$V_f = 0$ @ $r \to \infty$

$\therefore K_f = 0$

$E_f = 0$

$U_f = 0 \quad (r \to \infty)$

$K_f = 0$

$E_f = 0$

Conserve energy

$U_i + K_i = U_f + K_f$

$$-\frac{k_e q Q}{d} + \frac{1}{2} m v_0^2 = 0$$

$$\boxed{\therefore v_0 = \sqrt{\frac{2 k_e q Q}{m d}}}$$

$U_i + K_i = U_f + K_f$

$$-\frac{G M_E \cancel{m}}{R_E} + \frac{1}{2} \cancel{m} v_0^2 = 0$$

$$\boxed{v_0 = \sqrt{\frac{2 G M_E}{R_E}}}$$