PHYS 121 Jan. 31, 2025

To do:

- complete HW5 on PL by Feb. 7 @ 23:59

- Pre-Lab #2 is optional. It introduces computational methods that can be used to solve problems in physics.
  - Complete it if interested/curious. It is not worth marks, but we will grade the submissions received.

- Hands-On bonus project proposals due Feb. 10 @ 23:59
  - Email me a discription of your idea. Groups of 2 maximum.

Last Time:

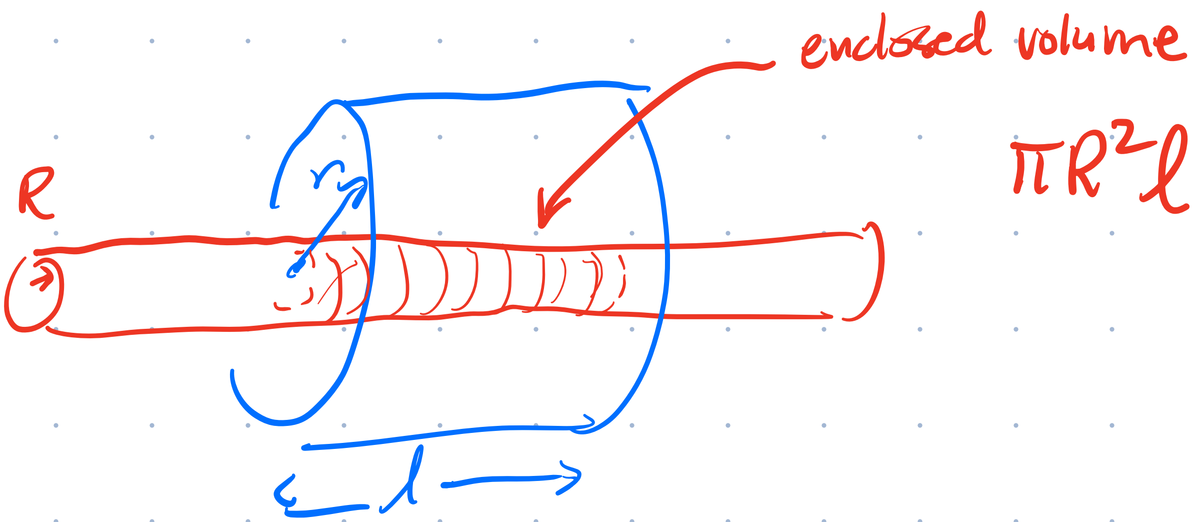
Find $\vec{E}$ due to the uniformly charged rod of radius R.

(a) Find $\vec{E}$ for $r > R$

(b) Find $\vec{E}$ for $r < R$.

$$\Phi = \oint \vec{E} \cdot d\vec{A} = E\,2\pi r \ell \quad \text{for both (a) \& (b)}$$

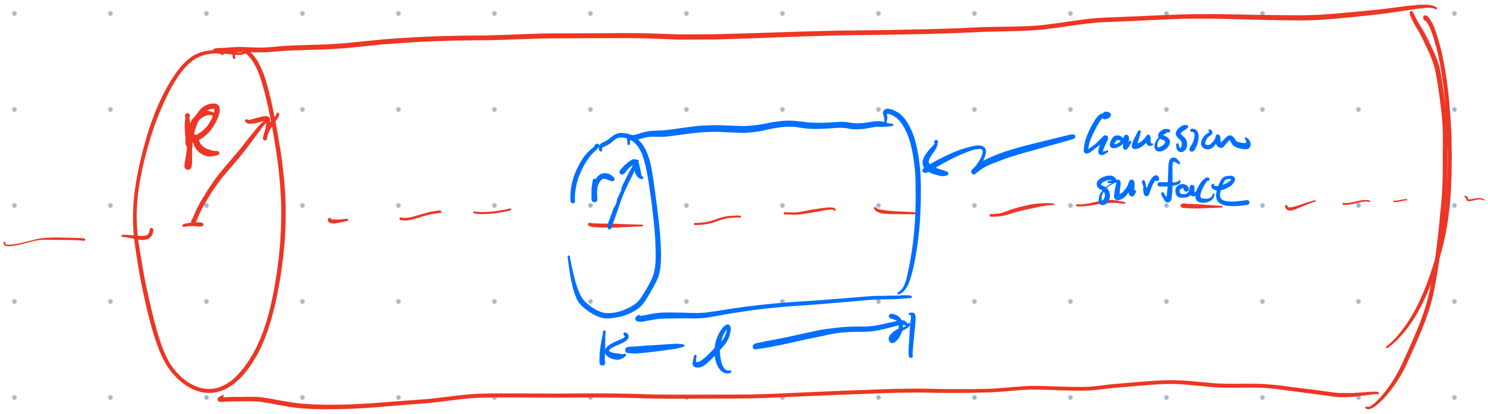
(a)



$$\Phi = \frac{q_{encl}}{\varepsilon_0} = \frac{\rho \pi R^2 \ell}{\varepsilon_0} \Rightarrow E = \frac{\rho \pi R^2}{2\pi \varepsilon_0 r}$$

$$= \frac{\lambda}{2\pi \varepsilon_0 r}$$

(b) $r < R$



Need to find q_{encl}...

Already know $\oint \vec{E} \cdot d\vec{A} = E(2\pi r l) = \Phi$

Now, find $q_{encl} = \rho\left(\begin{matrix}\text{volume of rod} \\ \text{contained in Gaussian surface}\end{matrix}\right)$

$$\frac{q_{encl}}{\varepsilon_0} \qquad \frac{\rho(\pi r^2 l)}{\varepsilon_0} = \Phi$$

equal

$$E(2\cancel{\pi}\cancel{r}\cancel{l}) = \frac{\rho(\cancel{\pi} r^{\cancel{2}} \cancel{l})}{\varepsilon_0}$$ solve for E.

$r < R$ $\quad \boxed{E = \dfrac{\rho r}{2\varepsilon_0}}$ ① E insider charged rod a dist r from the axis.

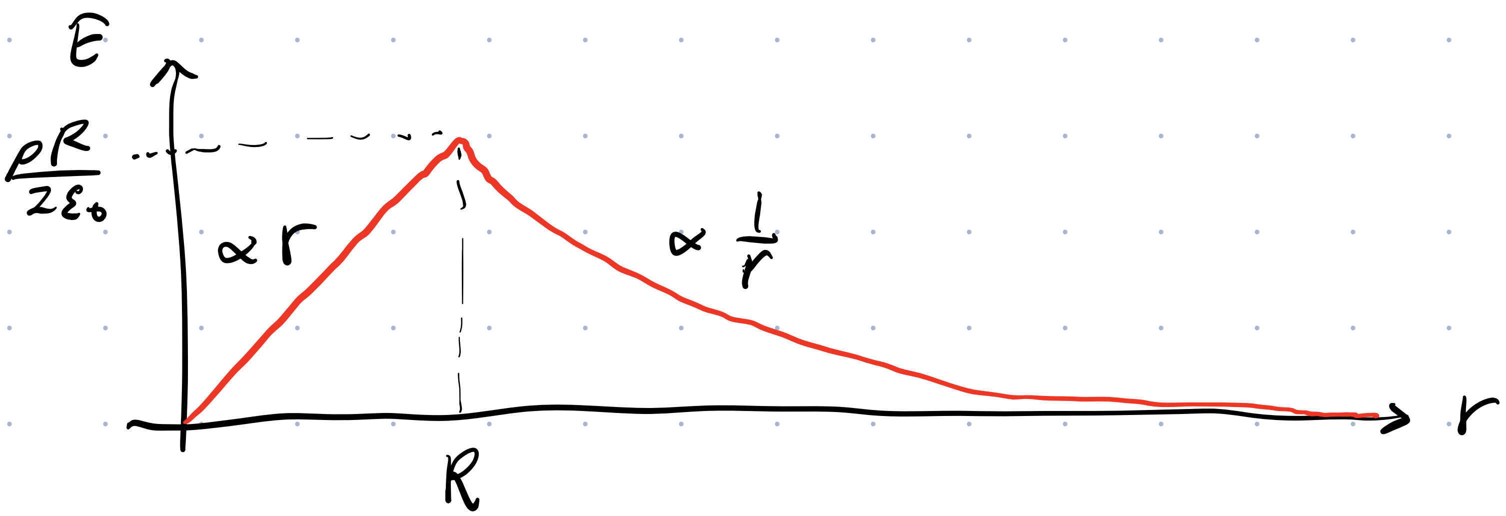
$r > R$ $\quad \boxed{E = \dfrac{\rho \cancel{\pi} R^2}{2\cancel{\pi}\varepsilon_0 r}}$ ② E outside rod a dist r from axis of rod.

If $r = R \rightarrow$ From ① $E = \dfrac{\rho R}{2\varepsilon_0}$

same ✓

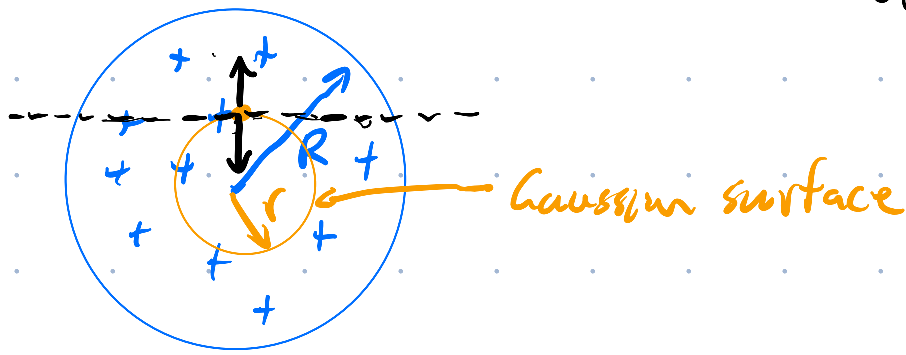
② $E = \dfrac{\rho R}{2\varepsilon_0}$

Plot E vs r for our charge rod of radius R.



End view of rod.

$$\Phi = \frac{q_{encl}}{\varepsilon_0}$$



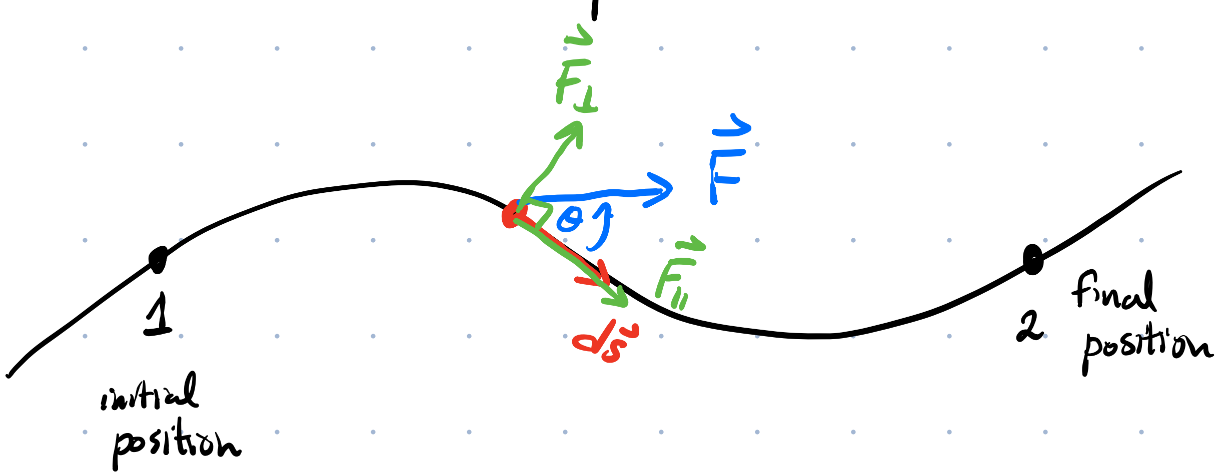
Chapter 7 OSUP v2

(Voltage)

Electric Potential & Electric Potential Energy.

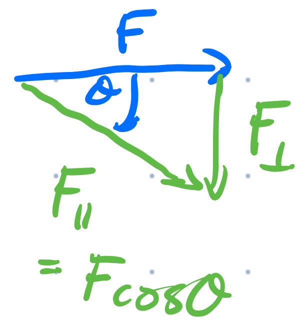
Recall the work-K.E. Theorem

$$\Delta K = W = \int_1^2 \vec{F} \cdot d\vec{s}$$



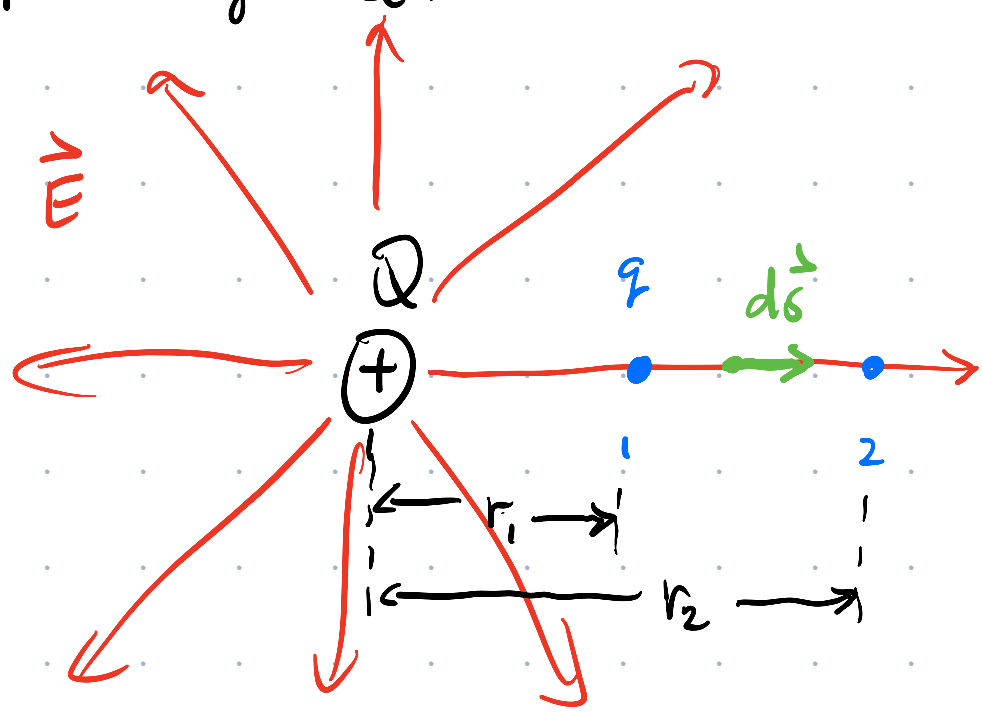
$$\vec{F}\cdot d\vec{s} = F\,ds\cos\theta = \underbrace{(F\cos\theta)}_{F_{\parallel}}\,ds$$

$$\vec{F}\cdot d\vec{s} = F_{\parallel}\,ds$$



Only the component of $\vec{F}$ that is $\parallel$ to the displacement $d\vec{s}$ contributes to the work W.

Apply work-K.E. theorem to a pt. charge q moving in an electric field due to another pt. charge Q.



Calc. the work along path from 1 to 2.
(straight line path)

$$\vec{E} \parallel d\vec{s} \Rightarrow \vec{E}\cdot d\vec{s} = E\,ds\underbrace{\cos 0}_{1} = E\,ds$$

Our displacement is in the radial dir'n.
so $d\vec{s} = d\vec{r}$

Know $\vec{E} = \frac{k_e Q}{r^2}\hat{r}$

Force on q in $\vec{E}$ is $\vec{F} = q\vec{E}$

$$= \frac{k_e q Q}{r^2}\hat{r}$$

$$W = \int_1^2 \vec{F}\cdot d\vec{r} = \int_1^2 \left(\frac{k_e q Q}{r^2}\hat{r}\right)\cdot\left(dr\,\hat{r}\right)$$

$$= \int_1^2 \frac{k_e q Q}{r^2}\,dr$$

$$= k_e q Q \int_{r_1}^{r_2} \frac{dr}{r^2}$$

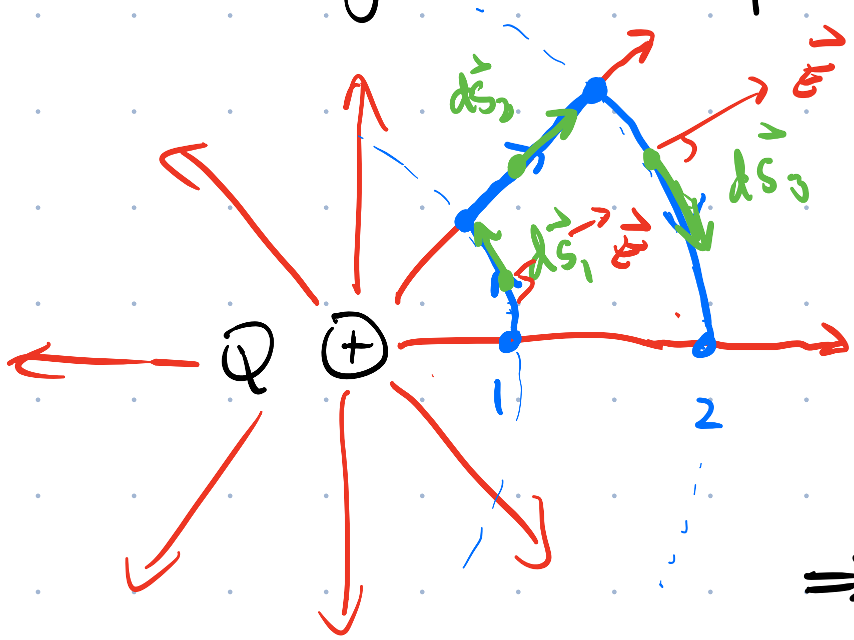
$$\therefore W = -k_e q Q \left. \frac{1}{r} \right|_{r_1}^{r_2}$$

$$\Delta K = W = -k_e q Q \left[\frac{1}{r_2} - \frac{1}{r_1} \right]$$

if q & Q are the same sign, then $\Delta K > 0$ & the K.E. increases as expected.

$$\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = -k_e q Q \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

Repeat the calc. of the work between pts 1 & 2, but along a different path.



For parts 1 & 3 of this path, $d\vec{s} \perp \vec{E}$

$\therefore \vec{F} \cdot d\vec{s} = 0$

$\Rightarrow$ Only part 2 of this path contributes to W.

$$W = \cancelto{0}{\int_{①} \vec{F}\cdot d\vec{s}} + \underbrace{\int_{②} \vec{F}\cdot d\vec{s}}_{\text{same as perious calculation.}} + \cancelto{0}{\int_{③} \vec{F}\cdot d\vec{s}}$$

$$W = \Delta K = -k_e q Q \left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Exactly the same as before.

No matter which path we choose from 1 to 2, the work will always be the same. The work is independent of that path taken & depends only on the starting & ending pts.

Forces for which the work is path indep. are called <u>conservative forces</u>. For conservative forces, can define a potential energy.

Conservation of mechanical energy requires:

change in potential energy.

$$\Delta K + \Delta U = 0$$

$$\Rightarrow \quad \Delta K = -\Delta U$$

So for our two point charges

$$\Delta K = -k_e q Q \left(\frac{1}{r_2} - \frac{1}{r_1} \right) = -\Delta U$$

$$= -(U_2 - U_1)$$

$$\Downarrow$$

$$\frac{k_e q Q}{r_2} - \frac{k_e q Q}{r_1} = U_2 - U_1$$

∴ The potential energy U associated w/ a pair of pt. charges Q & q is:

$$\boxed{U = \frac{k_e q Q}{r}}$$