PHYS 121

Jan. 31, 2025

To do:

- complete HW4 on PL by $\boxed{\text{today}}$ @ 23:59

- Pre-Lab #2 is optional. It introduces computational methods that can be used to solve problems in physics.
  - Complete it if interested/curious. It is not worth marks, but we will grade the submissions received.

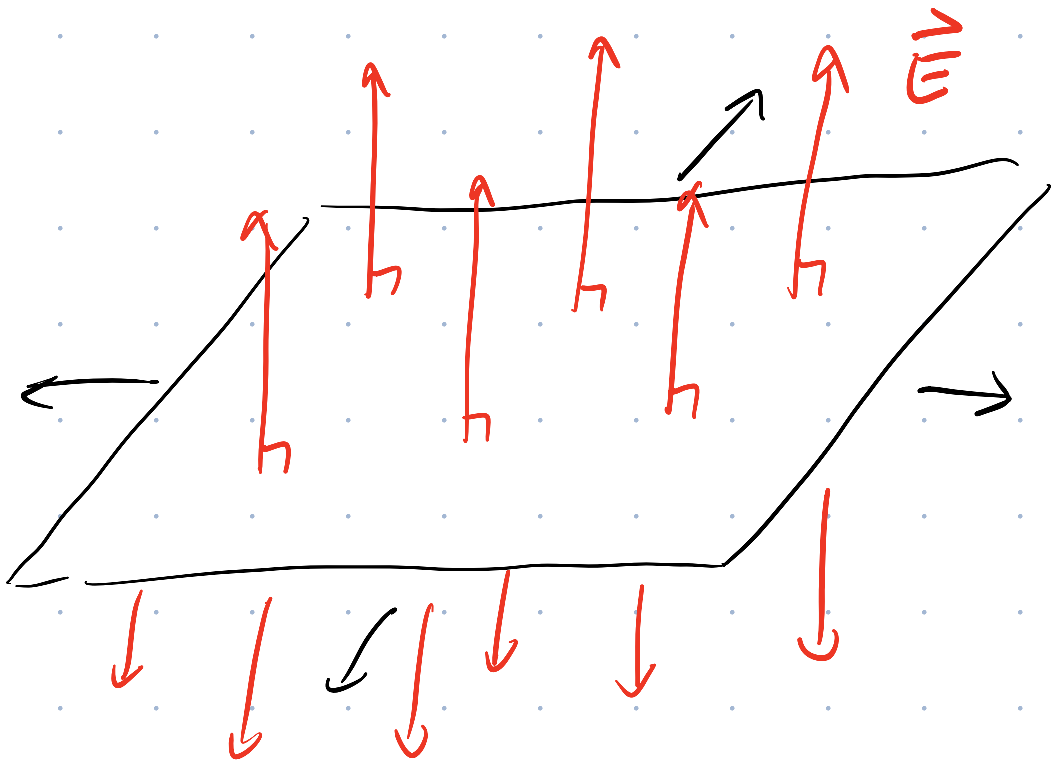
- Hands-On bonus project proposals due Feb. 10 @ 23:59
  - Email me a discription of your idea. Groups of 2 maximum.

Last Time: $\vec{E}$ due to a uniformly-charged infinite sheet w/ charge per unit area $\sigma$

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{end}}{\varepsilon_0} \qquad \text{Gauss's Law}$$

Calculate $\Phi$ in two ways:

$$\left.\begin{aligned} \Phi &= \oint \vec{E} \cdot d\vec{A} = 2EA \\ \Phi &= \frac{q_{end}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \end{aligned}\right\} \quad \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

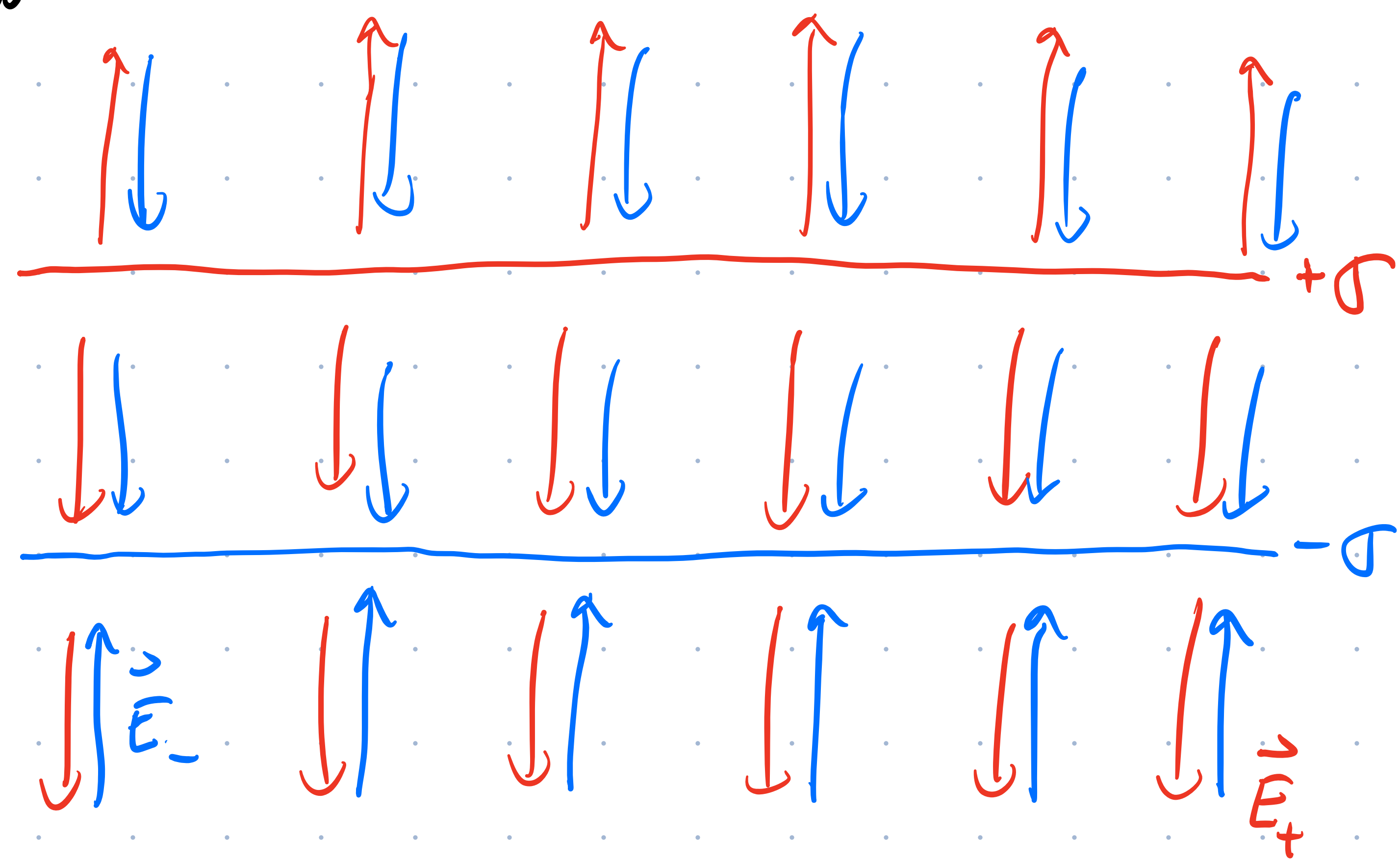


Today: Application of uniformly-charged sheets

$\Rightarrow$ Parallel-Plate Capacitor.

Consider two parallel sheets with uniform charge per unit area $\rightarrow$ sheets have opp. sign of charge.

side view




- Start by drawing $\vec{E}$ due to the positive sheet.
- Next, draw $\vec{E}$ due to the negative sheet.

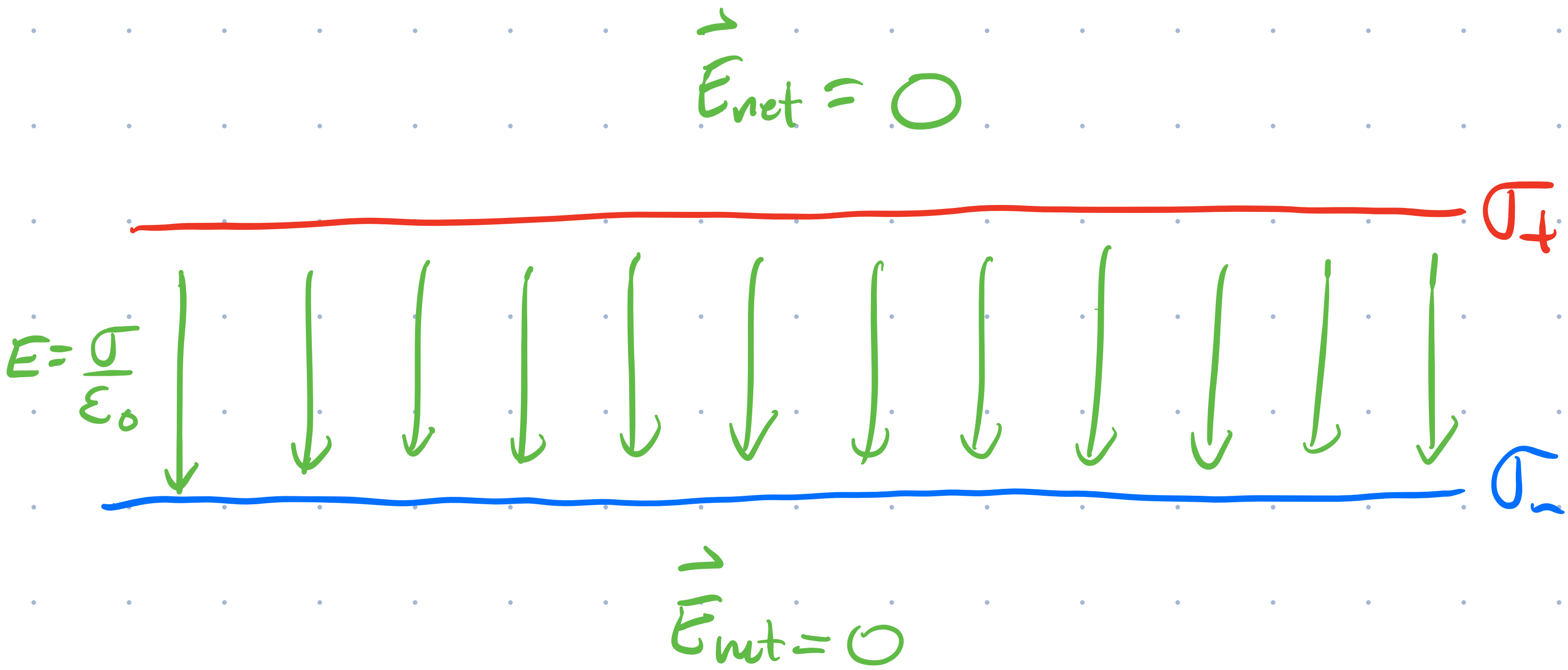
Note: $|\vec{E}_+| = \dfrac{\sigma}{2\varepsilon_0}$ $\quad |\vec{E}_-| = \dfrac{\sigma}{2\varepsilon_0}$

Above & below the parallel sheets/plates of the capacitor, $\vec{E}_+$ & $\vec{E}_-$ are in opp. dir'ns & sum to zero.

Between the two plates $\vec{E}_+$ & $\vec{E}_-$ add

$$|\vec{E}_{net}| = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$$ Between the plates.

Redraw the parallel plate cap. & the net $\vec{E}$.



- Parallel sheets provide a very easy & practical way of creating a uniform electric field w/ $\vec{E}=0$ everywhere except the region between the plates.
- We will see later that capacitors can also be used as charge-storage devices in electronic circuits.

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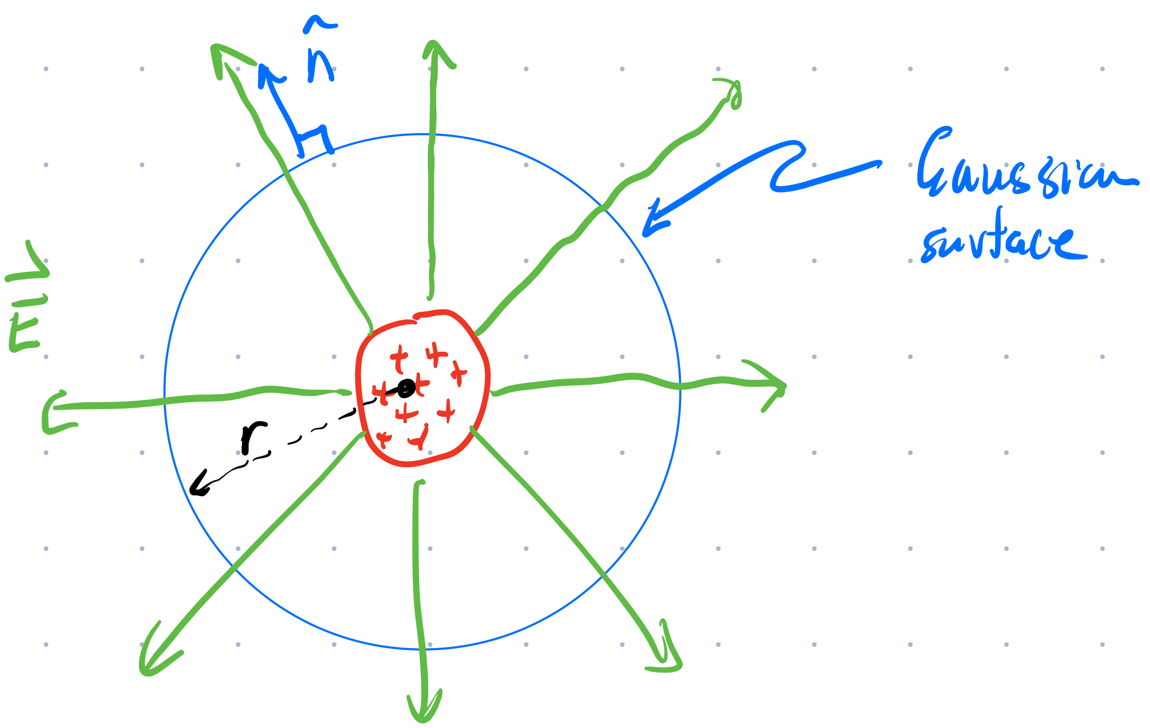
Gauss's Law is easy to apply in 3 scenarios

① Pt. charge or a spherical dist'n of charge at the centre of a spherical Gaussian surface.



- Here, $d\vec{A} \parallel \vec{E}$ on Gaussian surface

$$\vec{E}\cdot d\vec{A} = E\,dA$$

- $\vec{E}$ has a const. mag. everywhere on Gaussian surface.

$$\Phi = \oint E\,dA = E\oint dA = EA_{sphere}$$
$$= E(4\pi r^2)$$

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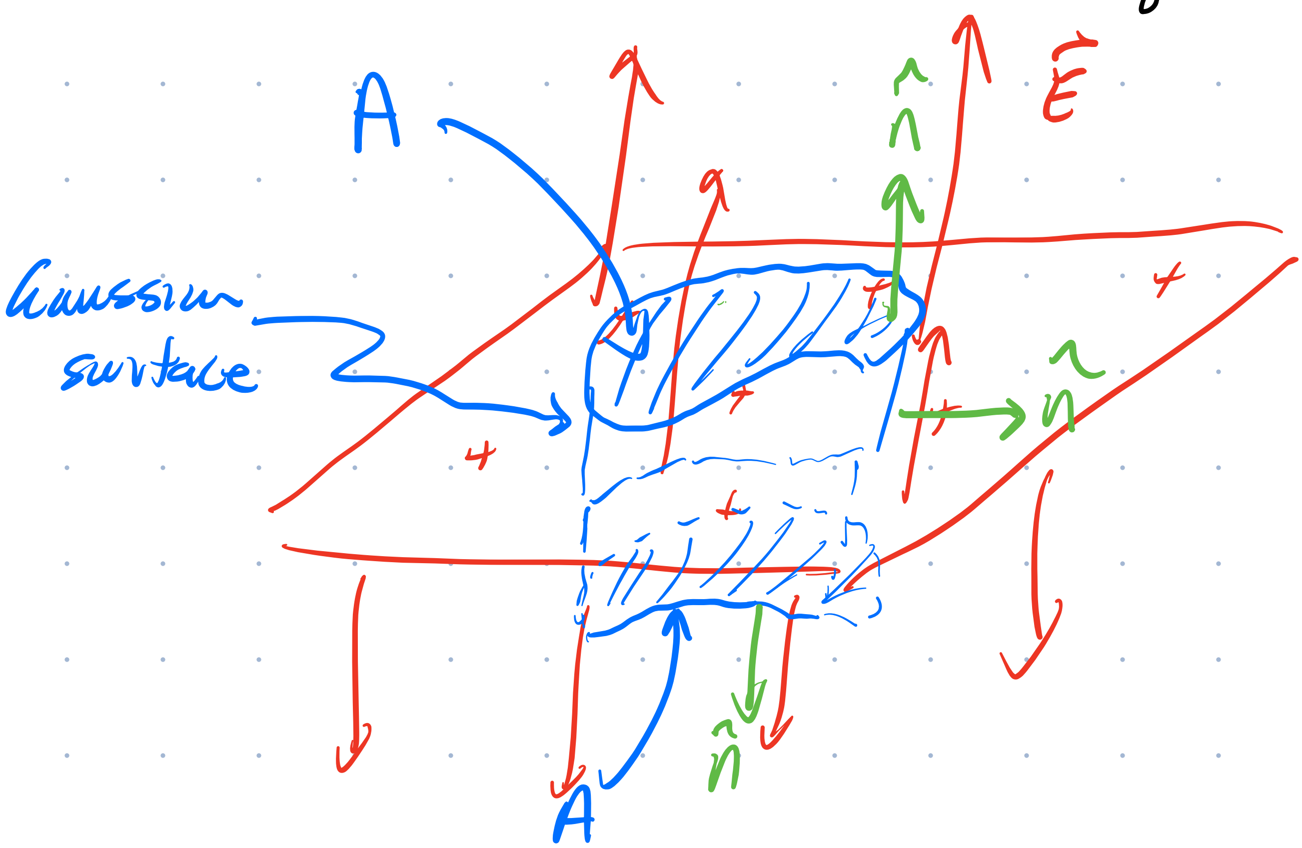
② Flat & uniform dist'n of charge.



Pick a Gaussian surface in which $\hat{n}$ is always either $\parallel$ or $\perp$ to $\vec{E}$

$$\oint \vec{E}\cdot d\vec{A} = \int_{top} \vec{E}\cdot d\vec{A} + \int_{btm} \vec{E}\cdot d\vec{A} + \cancelto{0}{\int_{side} \vec{E}\cdot d\vec{A}}$$

$\vec{E}\cdot d\vec{A} = E\,dA$ top & btm b/c $\hat{n} \parallel \vec{E}$

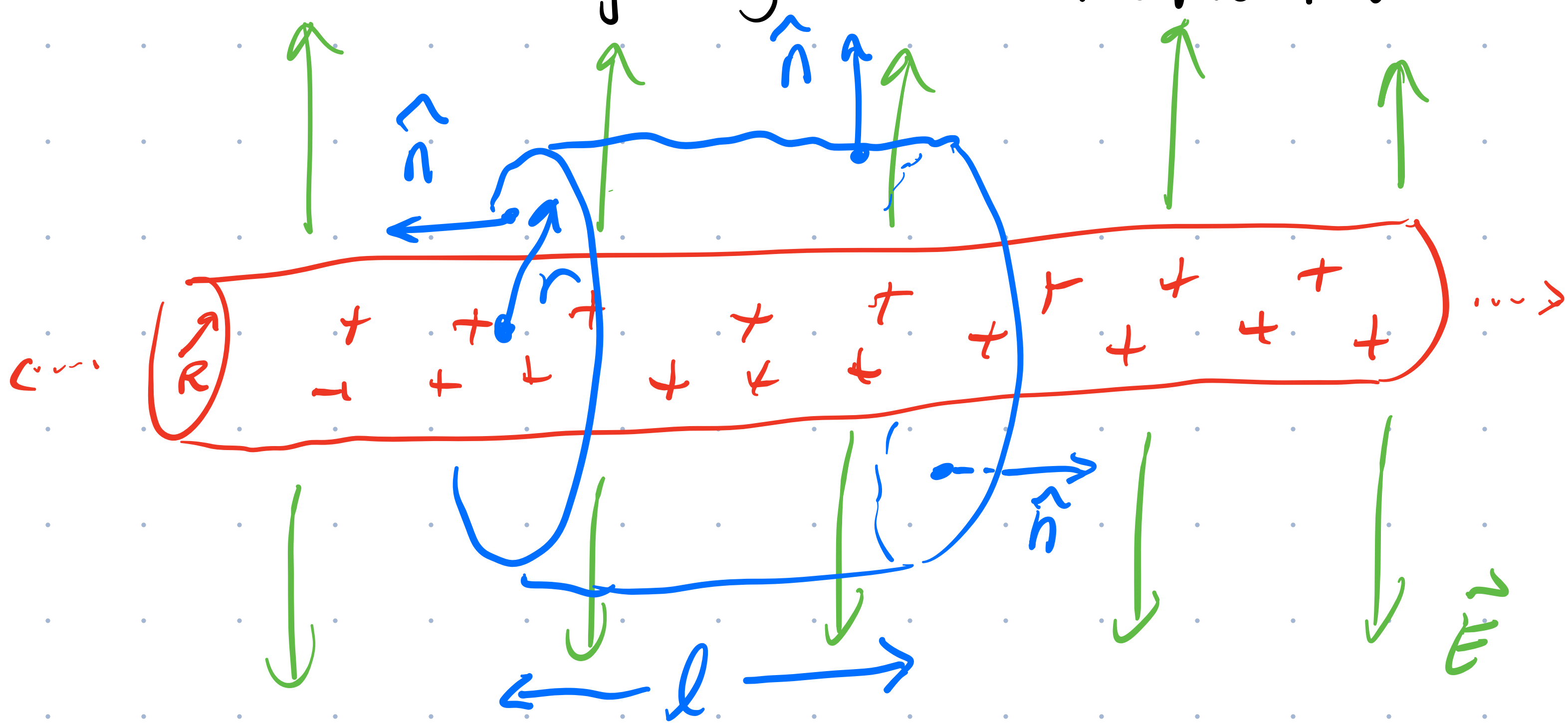
$\vec{E}\cdot d\vec{A} = 0$ side b/c $\hat{n} \perp \vec{E}$

Since $E$ is const, it can be pulled outside integrals for top & btm parts.

$$\Phi = \oint \vec{E}\cdot d\vec{A} = E\underbrace{\int_{top} dA}_{A} + E\underbrace{\int_{btm} dA}_{A} = 2EA$$

③ Cylindrical dist'n of charge.

→ very long wire of radius $R$.




By symmetry, $\vec{E}$ must be $\perp$ to charged wire.

Pick a Gaussian s.t. $\vec{E}$ is either $\parallel$ or $\perp$ to <u>all</u> parts of the closed surfaced.

A cylindrical Gaussian surface has 3 parts

- curved part w/ $\hat{n} \parallel \vec{E}$ s.t. $\vec{E} \cdot d\vec{A} = E\,dA$
- left end and right end for which $\hat{n} \perp \vec{E}$ and $\vec{E} \cdot d\vec{A} = 0$.

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \underbrace{\int_{\text{curved}} \vec{E}\cdot d\vec{A}}_{\int_{\text{curved}} E\,dA} + \cancel{\int_{\text{left}} \vec{E}\cdot d\vec{A}}^{\,0} + \cancel{\int_{\text{right}} \vec{E}\cdot d\vec{A}}^{\,0}$$

Everywhere on curved surface, same dist. $r$ from the centre of the charge dist'n.

$\therefore$ $\vec{E}$ has the same mag. everywhere on curved surface. $\rightarrow$ const.

$$\Phi = \oint \vec{E}\cdot d\vec{A} = E\int_{\text{curved}} dA = E\,A_{\text{curved}}$$

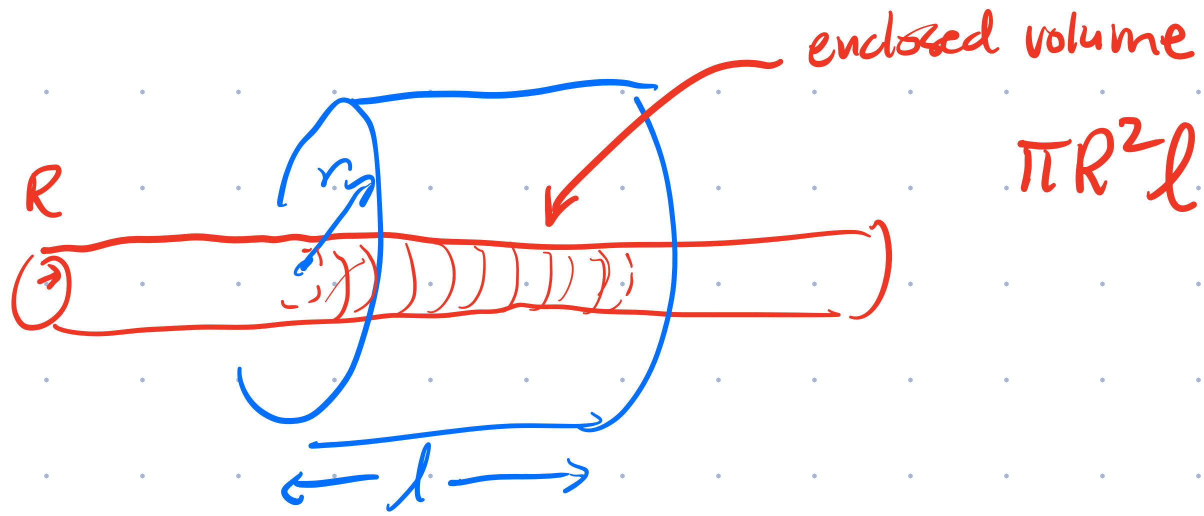
$$\boxed{\Phi = E(2\pi r l)}$$

Find $\vec{E}$ due to the uniformly charged rod of radius $R$.

(a) Find $\vec{E}$ for $r > R$

(b) Find $\vec{E}$ for $r < R$.

(a)




$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$$

Already worked out that

$$\Phi = \oint \vec{E} \cdot d\vec{A} = E(2\pi r l) \quad ①$$

we now need to find $\Phi = \dfrac{q_{encl}}{\varepsilon_0}$

Assume the charge per unit volume of the wire is $\rho$.

$$q_{encl} = \rho \left( \begin{array}{c} \text{volume of wire enclosed} \\ \text{by Gaussian surface} \end{array} \right)$$

$$= \rho(\pi R^2 l)$$

$$\therefore \Phi = \frac{q_{encl}}{\varepsilon_0} = \frac{\rho \pi R^2 l}{\varepsilon_0} \quad ②$$

Set ① = ②

$$E\, 2\pi r \cancel{l} = \frac{\rho \pi R^2 \cancel{l}}{\varepsilon_0}$$

solve for $E$

$$E = \frac{(\rho \pi R^2)}{2\pi \varepsilon_0 r}$$

Note that $\rho \pi R^2 = \lambda$ charge per unit length of the wire

$$\boxed{E = \frac{\lambda}{2\pi \varepsilon_0 r}}$$

long charged rod w/ $r > R$.

The same result we had for a long thin wire earlier in the term.