PHYS 121 Jan. 27, 2025

To do:

- complete HW4 on PL by Jan. 31 @ 23:59

- Pre-Lab #2 is optional. It introduces computational methods that can be used to solve problems in physics.
    - Complete it if interested/curious. It is not worth marks, but we will grade the submissions received.

- Quiz #1 will be on Wed. Jan. 29
    - See course website for details, including the formula sheet.

Last Time: Electric flux through a closed surface:

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

① any $\vec{E}$ due to charges outside a closed surface contribute zero flux, Φ.

② $\Phi > 0$ when surface encloses $q > 0$
$\Phi < 0$ " " " $q < 0$

③ Φ does <u>not</u> depend on position of q inside closed surface

④ Φ does <u>not</u> depend on shape of closed surface

⑤ Φ is proportional to the value of q enclosed by the surface.

The above observations led to Gauss's Law:

$$\boxed{\therefore \; \Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}}$$

Gauss's Law.

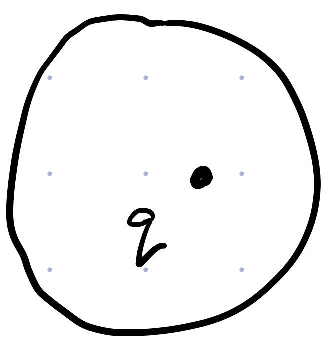
"encl" subscript reminds us that it is only the charge inside the closed surface that matters.

closed surface

<u>Today</u>: Applications of Gauss's law

Eg. What is Φ?

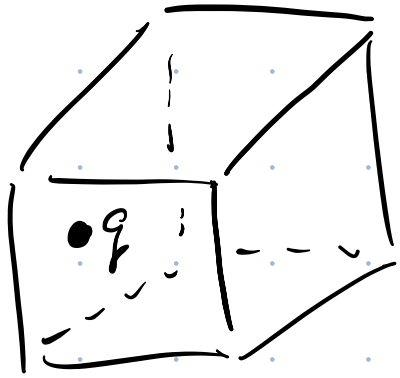
(a)



$q_{encl} = q$

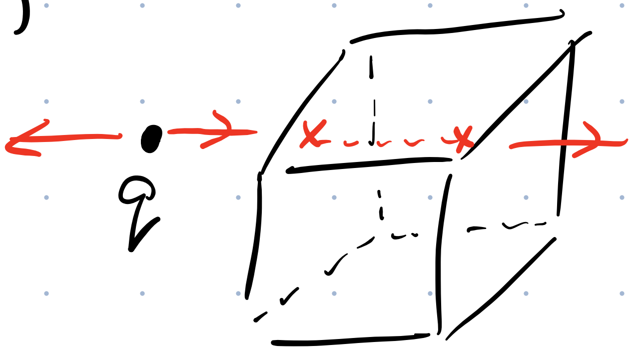
$$\therefore \Phi = \frac{q_{encl}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

(b)



$$\Phi = \frac{q}{\varepsilon_0}$$

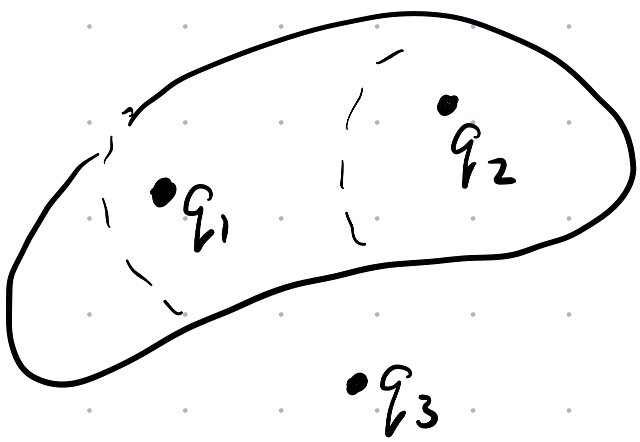
(c)



$$q_{encl} = 0$$

$$\therefore \Phi = \frac{q_{encl}}{\varepsilon_0} = 0$$

(d)



$$q_{encl} = q_1 + q_2$$

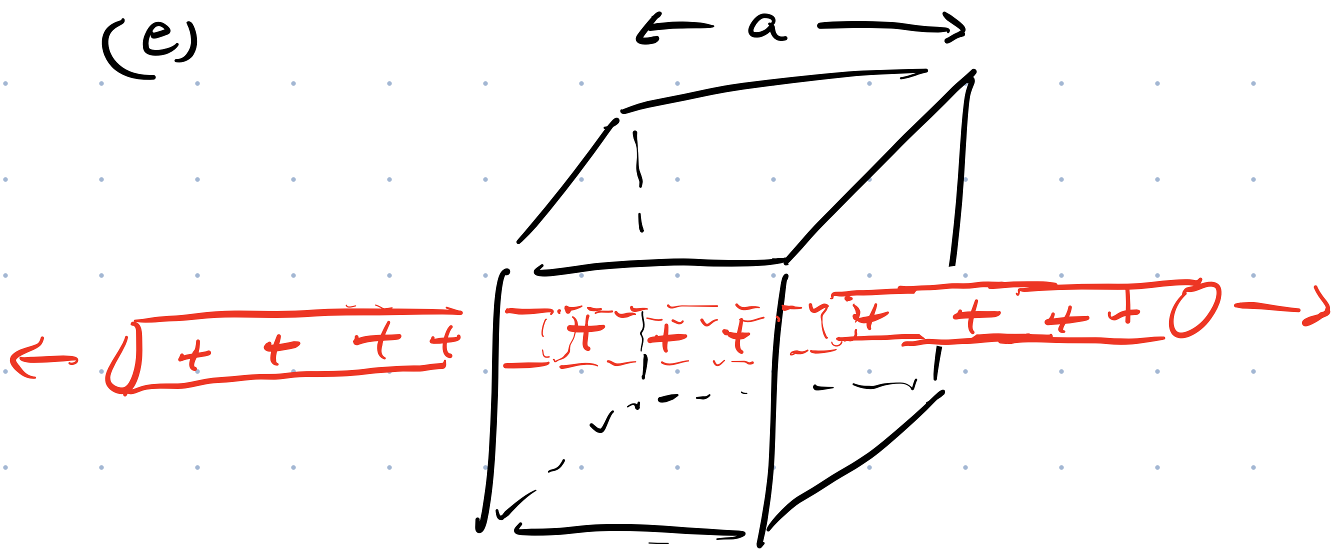
(q_3 is <u>outside</u> surface)

$$\therefore \Phi = \frac{q_{encl}}{\varepsilon_0} = \frac{q_1 + q_2}{\varepsilon_0}$$

Since we can build <u>any</u> distn of charge from a collection of pt. charges, Gauss's Law $\oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$ applies to <u>all</u>

charge distr'ns (not just pt. charges).

(e)



Long uniformly-charged rod passes through a cube of sides length a. The charge per unit length of the rod is λ. Find Φ through cube.

$$\Phi = \frac{q_{encl}}{\varepsilon_0}$$

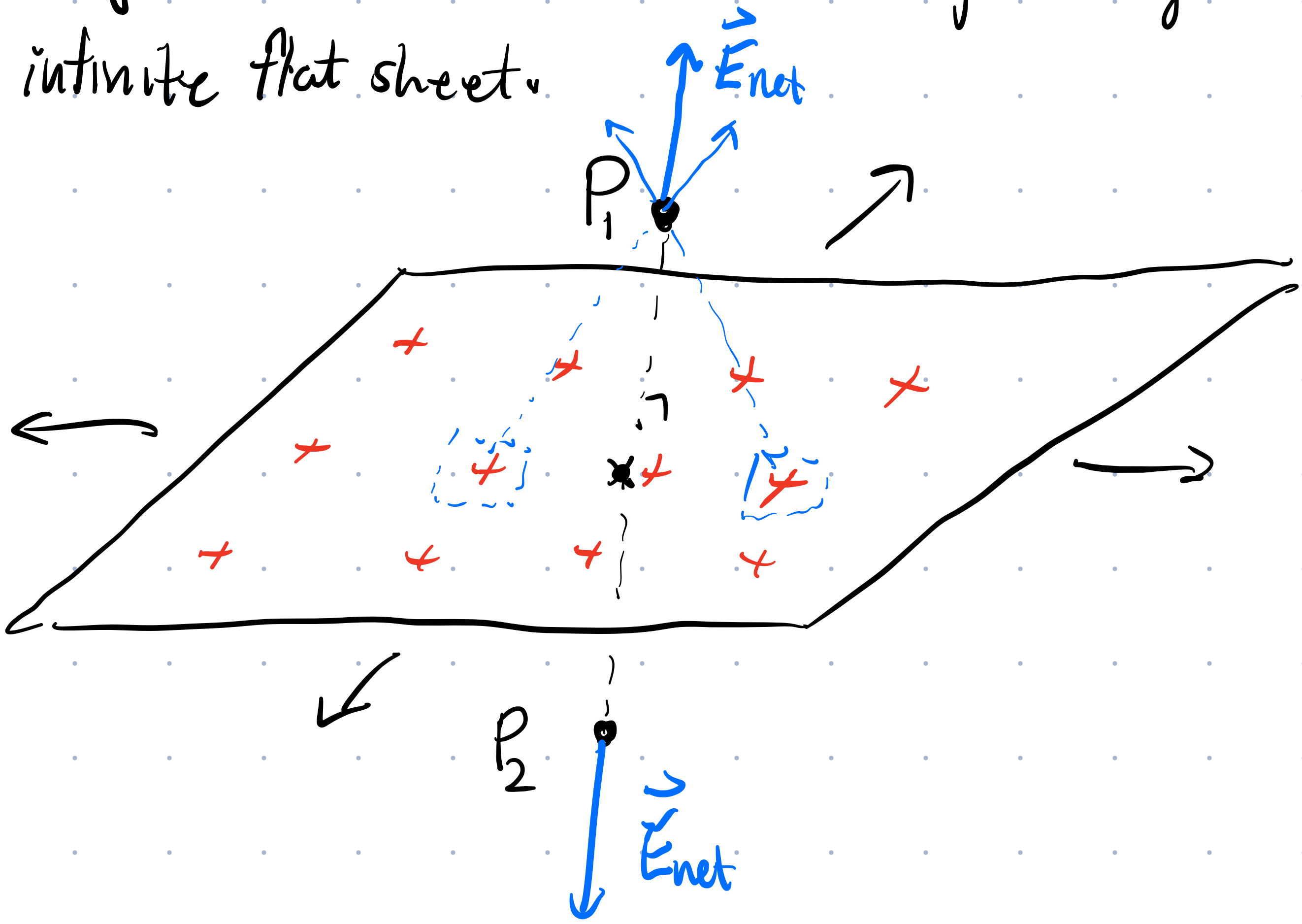
$q_{encl} = \lambda a$

since a length a of the rod is inside cube.

$$\boxed{\Phi = \frac{\lambda a}{\varepsilon_0}}$$

The main utility of Gauss's is to find electric fields due to certain continuous distributions of charge.

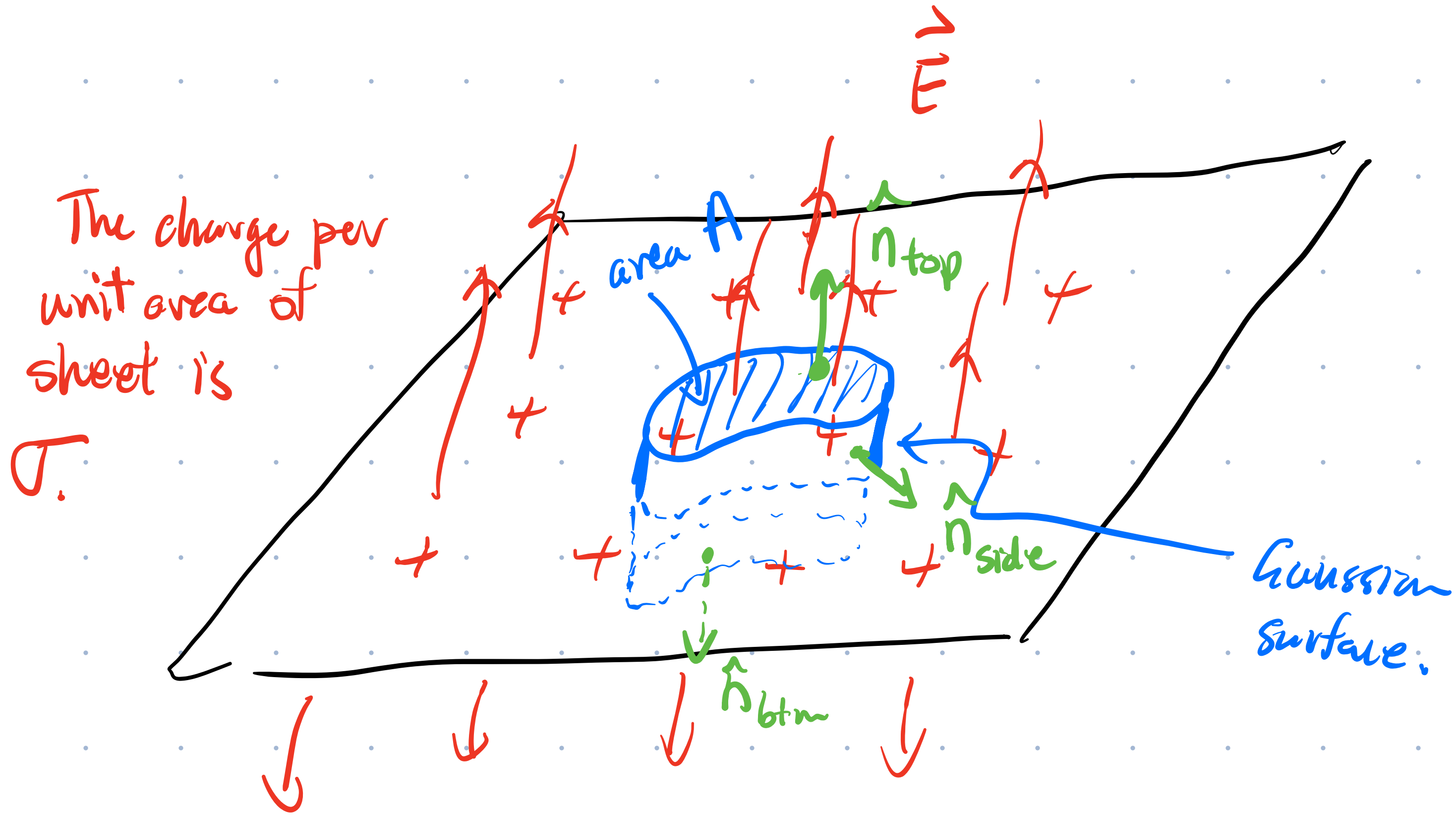
Eg. Find $\vec{E}$ due to a uniformly-charged infinite flat sheet.



Examine symmetry of this problem before applying Gauss's law. By symmetry only the vertical contributions to $\vec{E}$ will survive. $\therefore$ $\vec{E}$ due to charged sheet will be $\perp$ to the sheet.

To apply Gauss's Law, need to pick a closed surface to use to evaluate $\Phi = \oint \vec{E} \cdot d\vec{A}$.

This surface is called a Gaussian surface. To make the calculation simple, select a surface that matches the symmetry of $\vec{E}$ & the charge distn.



$\hat{n}_{top} \parallel \vec{E}$ $\qquad \vec{E} \cdot d\vec{A}_{top} = E\, dA_{top}$

$\hat{n}_{btm} \parallel \vec{E}$ $\qquad \vec{E} \cdot d\vec{A}_{btm} = E\, dA_{btm}$

$\hat{n}_{side} \perp \vec{E}$ $\qquad \vec{E} \cdot d\vec{A}_{side} = 0$

$$\oint \vec{E}\cdot d\vec{A} = \int \vec{E}\cdot d\vec{A}_{top} + \int \vec{E}\cdot d\vec{A}_{btm} + \underbrace{\cancelto{0}{\int \vec{E}\cdot d\vec{A}_{side}}}_{0}$$

closed surface

$$= \int E\, dA_{top} + \int E\, dA_{btm}$$

Since the top & btm parts of our Gaussian surface are parallel to the sheet, expect E to be const. on these parts of our Gaussian surface.

$$\Phi = \oint \vec{E}\cdot d\vec{A} = E\underbrace{\int dA_{top}}_{A} + E\underbrace{\int dA_{btm}}_{A}$$

$$= EA + EA = 2EA$$

$$\boxed{\Phi = 2EA} \quad (1)$$

Next, find Φ via $\frac{q_{encl}}{\varepsilon_0}$, where q_{encl} is the charge inside our Gaussian surface.

Since the charge per unit area of the sheet is σ & the cross-sectional area of the Gaussian surface is A:

$$q_{encl} = \sigma A$$

$$\boxed{\Phi = \frac{q_{encl}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}} \quad 2$$

We've now found Φ in two ways (1, 2) and they must be equal

$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{\therefore E = \frac{\sigma}{2\varepsilon_0}}$$

Electric field due to a uniformly-charged sheet. It is indep. of distance from the sheet.

Summary:

0-D	Pt. charge q.	$E = \frac{k_e q}{r^2}$	$E \propto \frac{1}{r^2}$
1-D	Line of charge λ.	$E = \frac{\lambda}{2\pi\varepsilon_0 r}$	$E \propto \frac{1}{r^1}$
2-D	Sheet of charge σ	$E = \frac{\sigma}{2\varepsilon_0}$	$E \propto \frac{1}{r^0}$