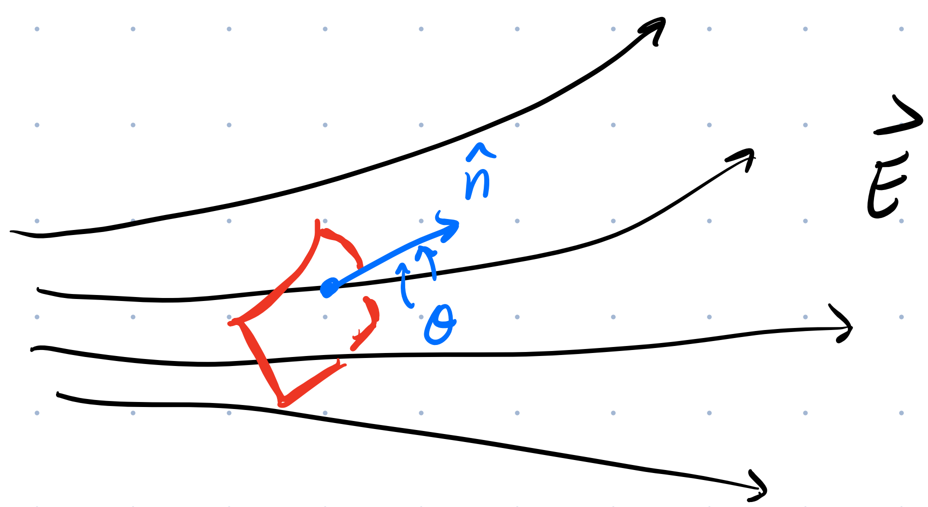
PHYS 121 Jan. 24, 2025

To do:

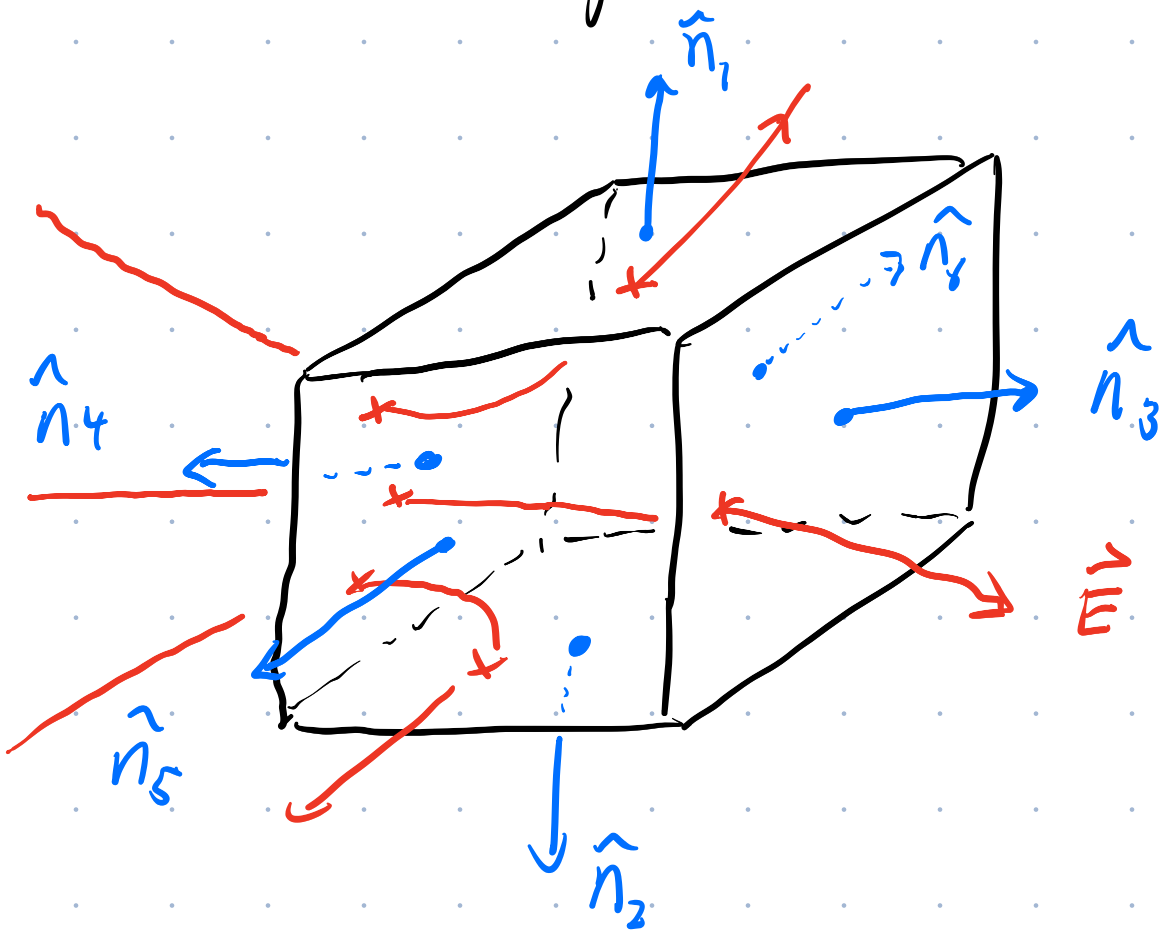
- complete HW 3 on PL by $\boxed{\text{today}}$ @ 23:59
- Complete Pre-Lab #1 before the start of Lab 1 next week
    - Complete Pre-labs independently → No partners
- Quiz #1 will be on Wed. Jan. 29
    - See course website for details, including the formula sheet.

Last Time:

Electric flux $\Phi = \int \vec{E} \cdot d\vec{A}$



# Electric flux through a closed surface



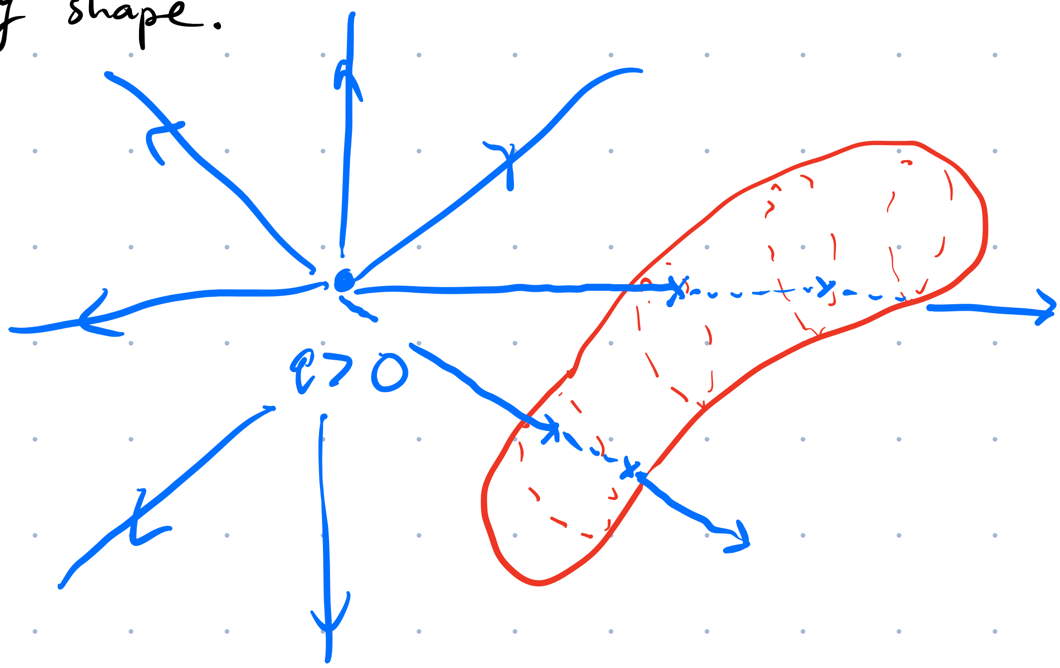

For a closed surface, define our normal vectors $\hat{n}$ such that they point <u>outwards</u>.

When $\hat{n}$ anti-parallel to $\vec{E}$, $\Phi < 0$ (neg)

When $\hat{n}$ parallel to $\vec{E}$, $\Phi > 0$ (pos)

For a closed surface w/ nothing inside, the net flux due to an <u>external</u> $\vec{E}$ is zero. Equal no. of $\vec{E}$ fields enter the closed surface (neg. flux) and exit the closed surface (pos. flux).

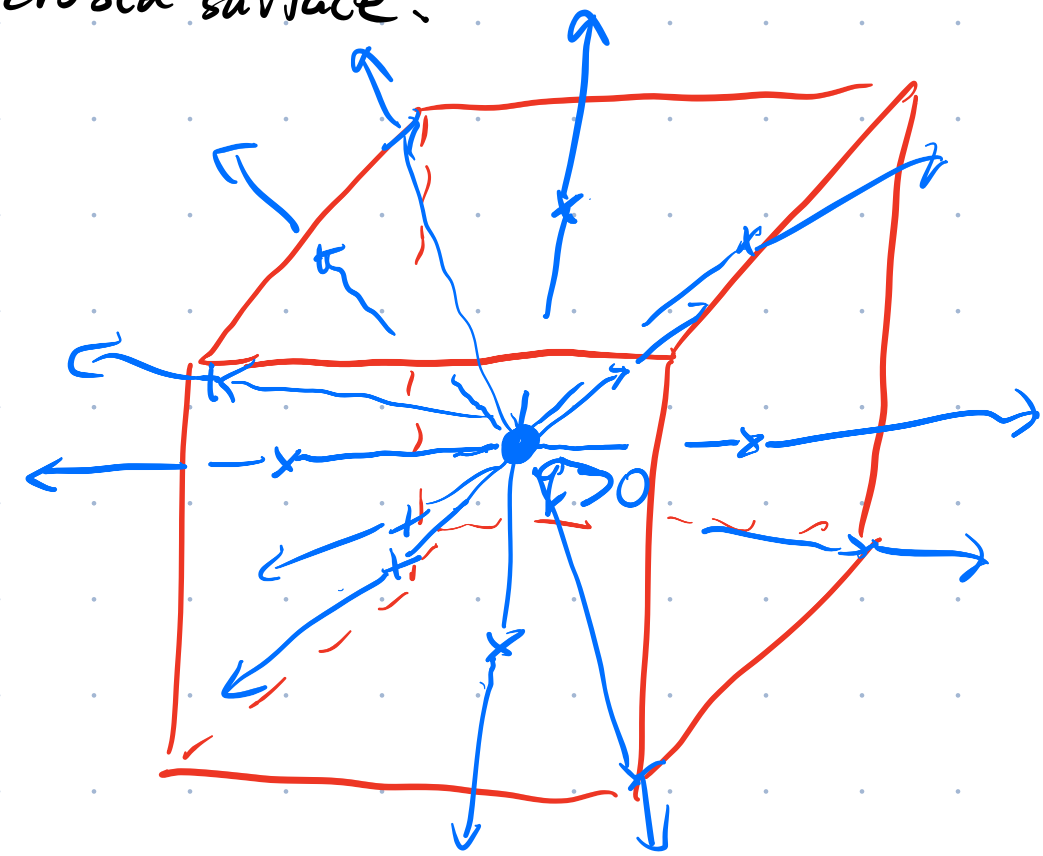
Think about a pt. charge next to a closed surface of any shape.




Since the same no. of $\vec{E}$-field lines enter and exit the surface, the net flux is zero.

Any charge dist'n outside a closed surface contrib. zero net flux.

Next, imagine placing a pt. charge inside a closed surface.



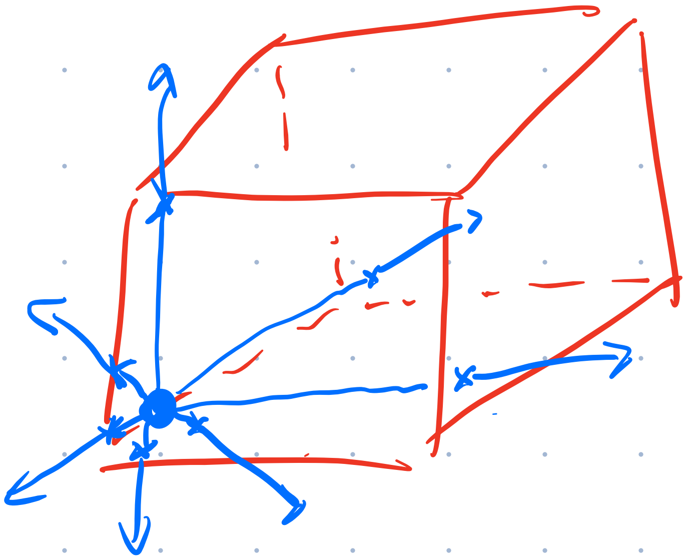

In this case, we only have $\vec{E}$-field lines exiting the box. → All contribute positive flux.

$$\Phi > 0 \quad \text{for} \quad q > 0$$

If we put in a $q<0$ (neg) inside the box, we would get only negative flux.

$$\Phi < 0 \quad \text{for} \quad q < 0$$

If we move our pt. charge from the centre of the box to one of the corners, we get:



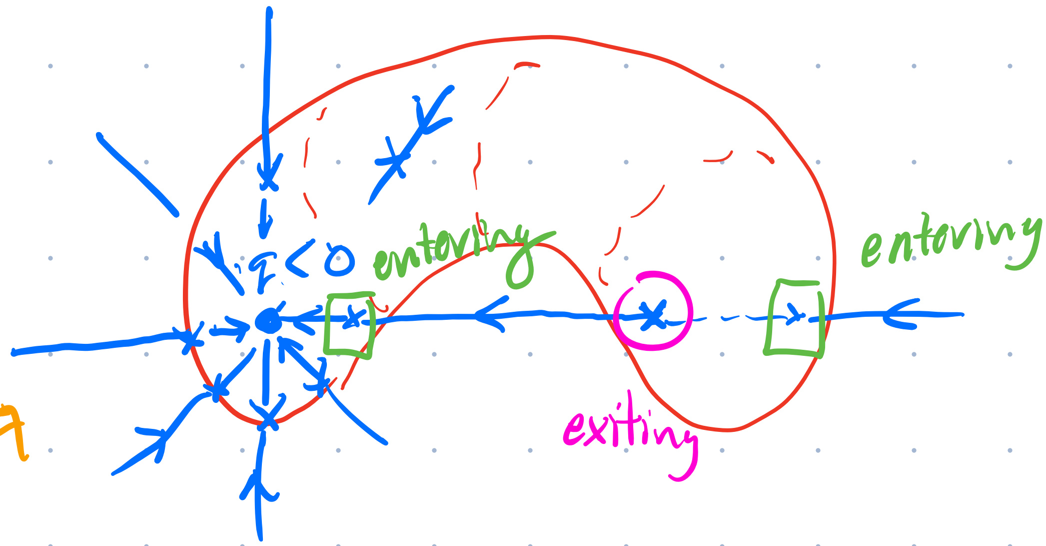
The total number of $\vec{E}$-field lines exits the closed surface does not change as we move the charge around inside. Always have same no. of lines exiting the surface. $\Phi$ is indep. of pos. of charge inside our closed surface.

If we double the value of $q$ in our closed surface, then $\vec{E}$ field doubles $\left( \vec{E} = \frac{k_e q}{r^2} \hat{r} \right)$

$\Rightarrow$ twice as many field lines exiting our surface.

$\Rightarrow$ $\Phi$ doubles.

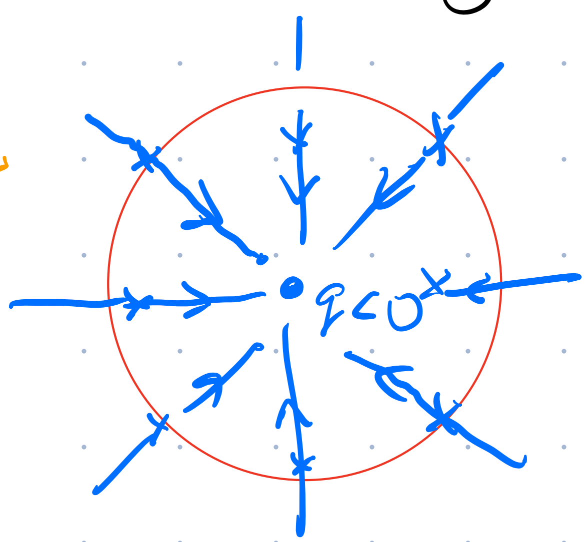
What if our surface had a funny shape?




If $q$ is the same, $\Phi$ is the same.

one exiting contribution is canceled by one "extra" entering contribution.

$\therefore$ flux through funny-shaped surface is identical to flux through any other closed surface containing the same charge.



Summary of Observations:

① any $\vec{E}$ due to charges outside a closed surface contribute zero flux, $\Phi$.

② $\Phi > 0$ when surface encloses $q > 0$
$\Phi < 0$ " " " $q < 0$

③ $\Phi$ does not depend on position of $q$ inside closed surface

④ $\Phi$ does not depend on shape of closed surface

⑤ $\Phi$ is proportional to the value of $q$ enclosed by the surface.

From these observations, we know

$$\Phi \propto q$$

we also know that $\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$

For a closed surface, add a O to the integral to remind ourselves to the surface is "closed" not "open".

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

closed surface surrounding a cavity.

$\Rightarrow$ For closed surfaces

$$\oint \vec{E} \cdot d\vec{A} \propto q \Rightarrow$$ Will become Gauss's Law.

Find flux through a closed surface containing a pt. charge.

- free to pick the shape of surface and position of pt. charge.

- Make choices that simplify the calculation of $\Phi = \oint \vec{E} \cdot d\vec{A}$

    - If $\vec{E} \parallel d\vec{A}$, then $\vec{E} \cdot d\vec{A}$

$$= E\,dA \underbrace{\cos 0}_{1}$$

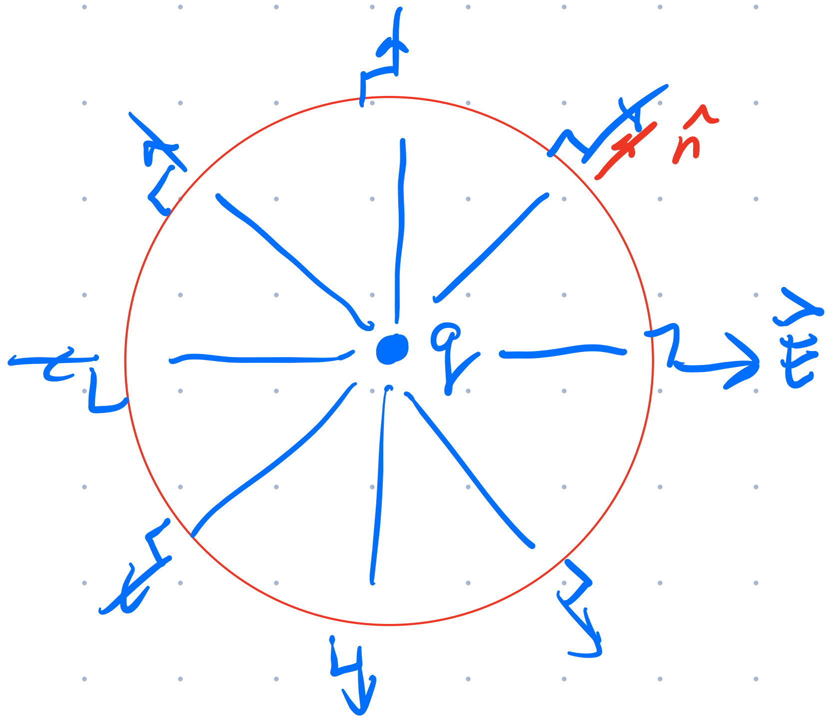
$$= E\,dA.$$

    - If $\vec{E} \perp d\vec{A}$, then $\vec{E} \cdot d\vec{A}$

$$= E\,dA \underbrace{\cos 90^\circ}_{0}$$

$$= 0$$

If $\vec{E}$ has a constant magnitude everywhere on surrounding surface, the we can take $E$ outside the integral.




Chose to surround $q$ w/ a sphere and put it at the centre.

$E$ is the same mag. everywhere on sphere ✓

$\vec{E}$ exits $\perp$ to surface. $\Rightarrow$ $\hat{n} \parallel \vec{E}$ ✓

$$\vec{E}\cdot d\vec{A} = E\,dA$$

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$ know $\vec{E} \cdot d\vec{A} = E\,dA$

$$= \oint E\,dA$$ know $E$ is const $\left(E = k_e \frac{q}{r^2}\right)$ everywhere on surface.

$$= E \oint dA$$ $\oint dA =$ surface area of sphere $= 4\pi r^2$

$$\therefore \Phi = E(4\pi r^2)$$ since $E$ is due to a pt. charge, sub in $E = k_e \frac{q}{r^2}$

$$\therefore \Phi = k_e \frac{q}{\cancel{r^2}} (4\pi \cancel{r^2}) = q 4\pi \underbrace{k_e}_{\frac{1}{4\pi\varepsilon_0}}$$

$$\Phi = \cancel{4\pi} \left(\frac{1}{\cancel{4\pi}\varepsilon_0}\right) q$$

$\boxed{\therefore \Phi = \frac{q}{\varepsilon_0}}$ for a closed surface containing charge $q$.

$$\boxed{\therefore\ \Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}}$$

Gauss's Law.