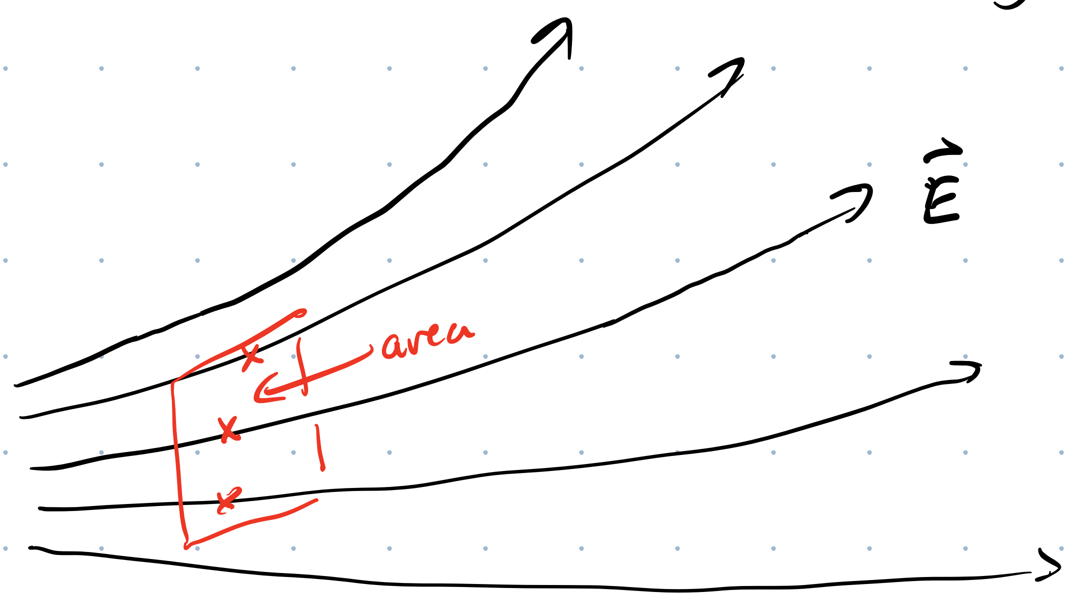
PHYS 121 Jan. 22, 2025

To do:

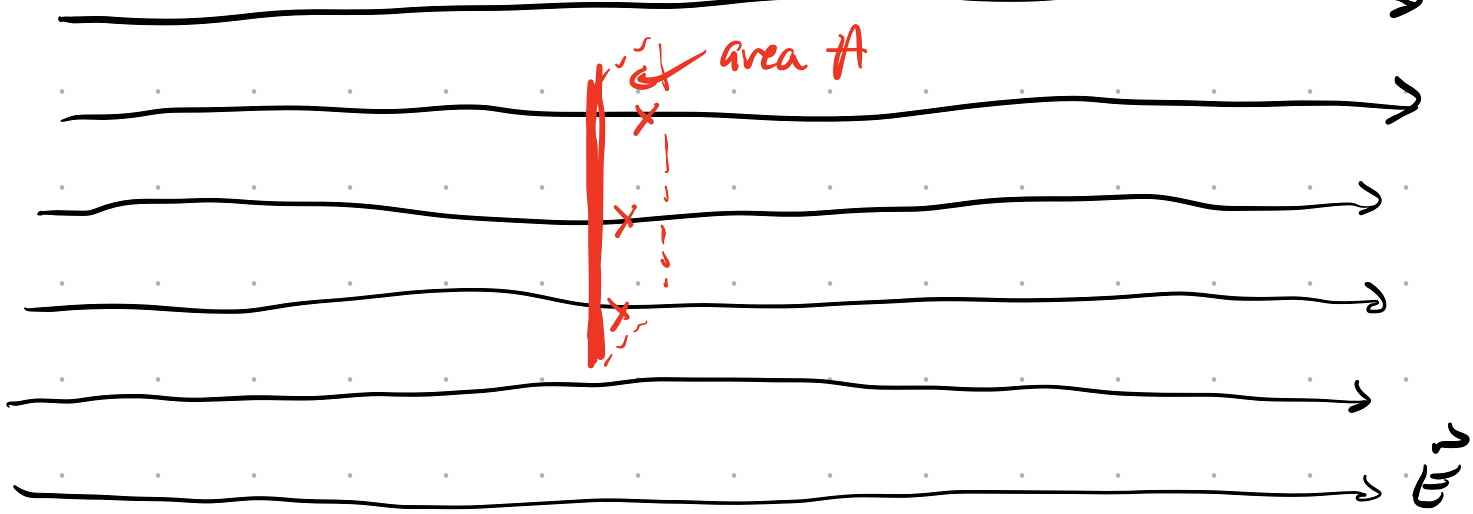
- complete HW 3 on PL by Jan. 24 @ 23:59
- Labs & Tutorials start $\boxed{\text{this week}}$
  - There is <u>no</u> pre-lab for the first lab. (Lab 0)
- Complete Pre-Lab #1 before the start of Lab 1 next week
  - Complete Pre-labs independently → No partners
- Quiz #1 will be on Wed. Jan. 29
  - See course website for details, including the formula sheet.

Last Time: Electric Flux Φ

The flux Φ through a surface is proportional to the number of field lines passing through that surface.

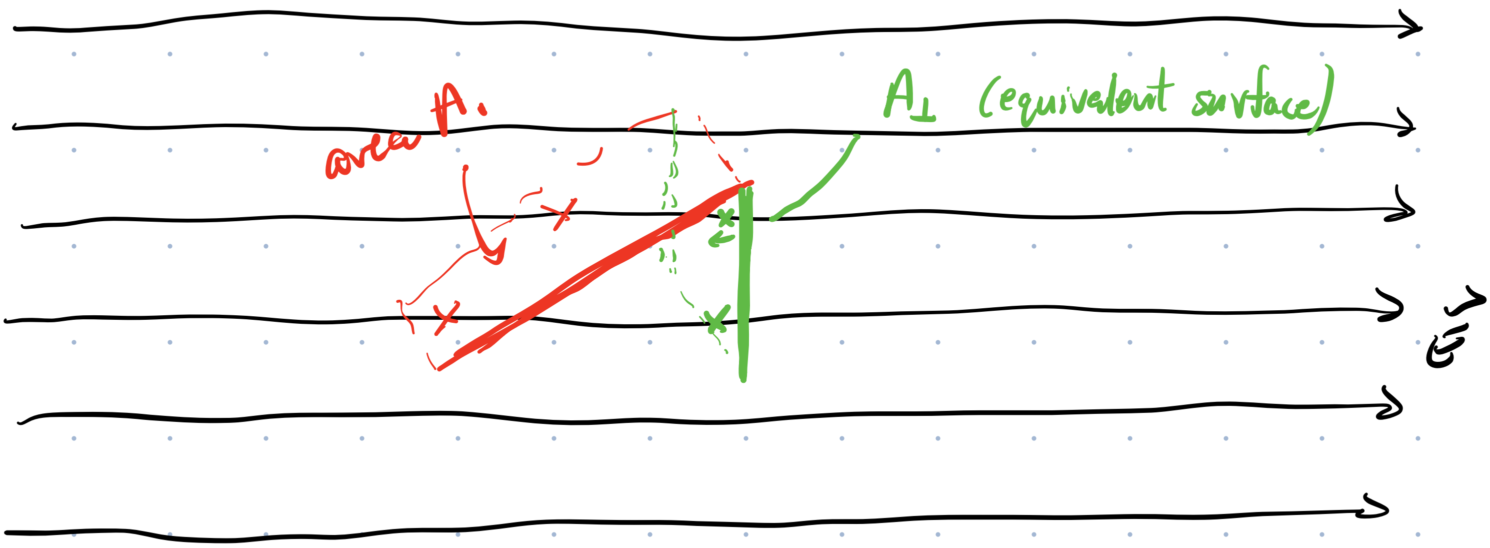


For a uniform electric field, with a surface $\perp$ to $\vec{E}$,



$$\Phi = EA$$

When the surface is tilted at an angle w.r.t. $\vec{E}$, fewer field lines pass through the surface and Φ is reduced:



We will define an "equivalent" surface that has the same flux through it. The green surface ($A_\perp$) has same flux (two lines) as the origonal red surface.

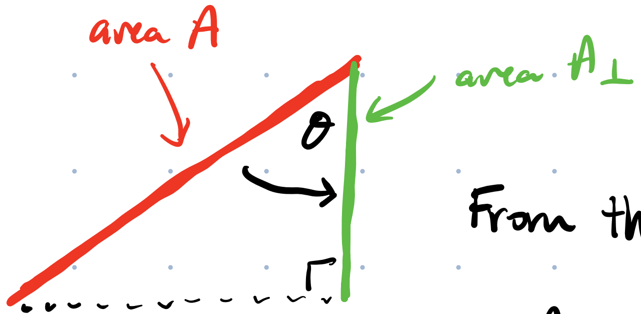
Since green surface is $\perp$ to a uniform $\vec{E}$, know:

$$\Phi = EA_\perp$$

flux for both red and green surface.

Relate $A_\perp$ to A

green red.



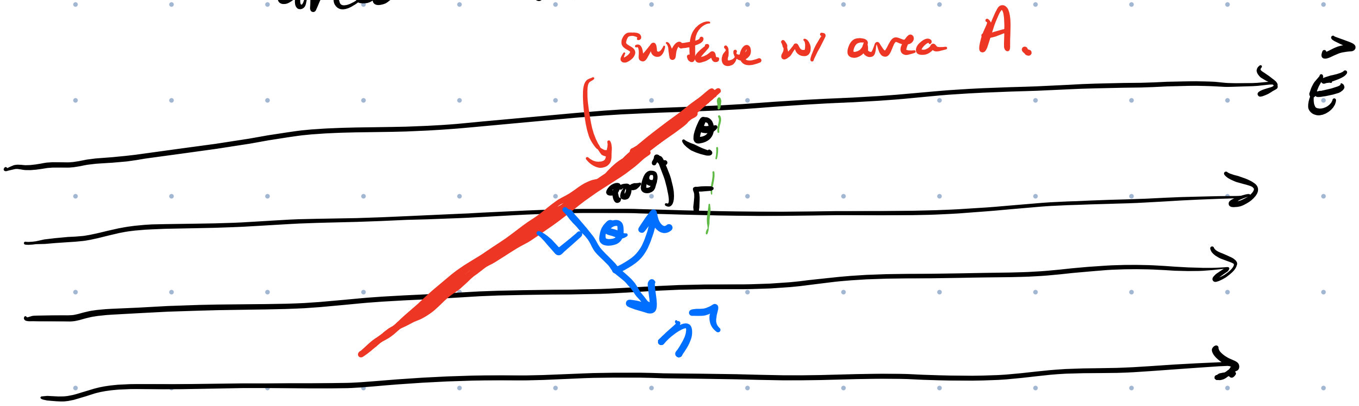
From this right-angle triangle

$$A_\perp = A\cos\theta$$

$$\therefore \Phi = EA_\perp = EA\cos\theta.$$

$\therefore$ $\Phi = EA\cos\theta$ is valid for a flat surface in a uniform electric field where the surface can make any angle w.r.t. $\vec{E}$.

Since $\vec{E}$ is a vector, we'd like to write Φ in terms of vector quantities. To do that, we need to define an area vector.



Define a unit vector $\hat{n}$ that is $\perp$ to our surface

$\hat{n}$ makes an angle θ w.r.t. $\vec{E}$.

Now, we can express area as a vector quantity

$$\vec{A} = A\hat{n} \qquad |\vec{A}| = A \text{ (area)}$$

dir'n of $\vec{A}$ is given $\hat{n}$.

Now cosider $\vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{n})$

$$= A\,\vec{E}\cdot\hat{n}$$

$$= A\underbrace{|\vec{E}|}_{E}\underbrace{|\hat{n}|}_{\substack{1 \\ \text{unit vector}}}\cos\theta$$

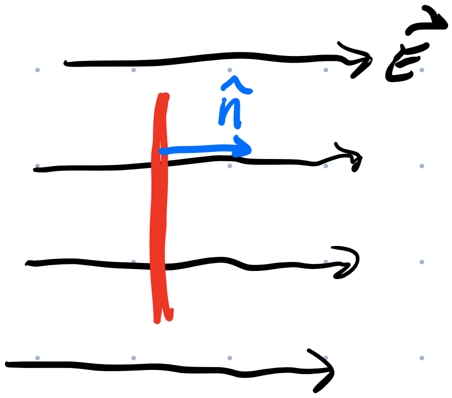
$\therefore \vec{E}\cdot\vec{A} = EA\cos\theta$ same expression we obtained for Φ.

$$\boxed{\therefore \Phi = \vec{E}\cdot\vec{A}}$$

a flat surface in a uniform $\vec{E}$ @ arbitrary angle θ.

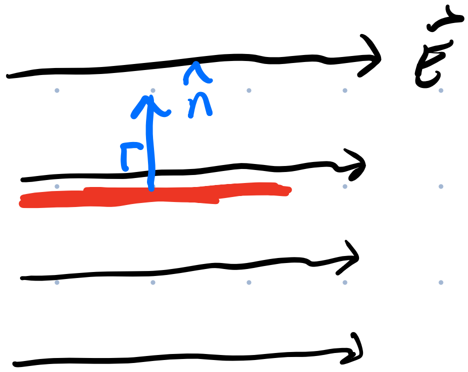
Test $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$ for some simple cases.

$\theta = 0$ case



$$\Phi = EA\underbrace{\cos 0}_{1} \Rightarrow \Phi = EA.$$

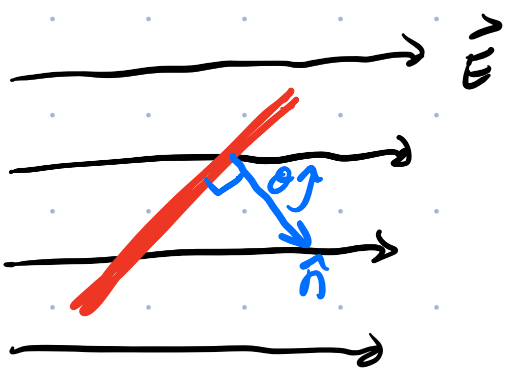
$\theta = 90^\circ$ case.



$$\Phi = EA\underbrace{\cos 90^\circ}_{0} = 0. \checkmark$$

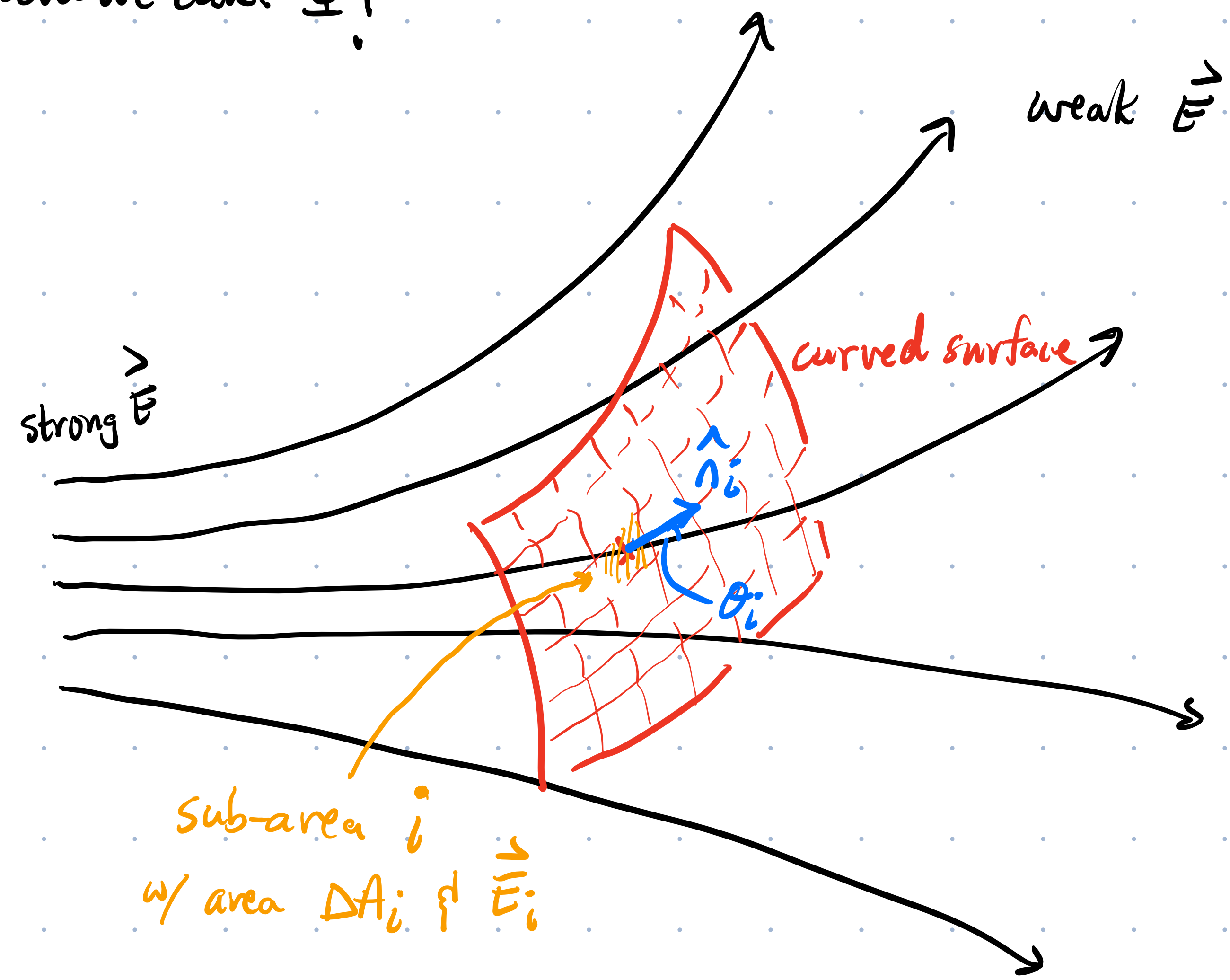
no flux crosses the surface when $\hat{n} \perp \vec{E}$.

$0 < \theta < 90^\circ$ case.



$$\boxed{\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta}$$

What if $\vec{E}$ is <u>not</u> uniform & our surface is <u>not</u> flat?
How can we calc. Φ?



To handle this case, we divide large area into small sub-areas. Pick the sub-areas to be small enough that they can be approximated as flat & $\vec{E}$ through each sub-area is approx. uniform.

For sub-area i, the flux is:

$$\Phi_i = E_i \Delta A_i \cos\theta_i$$

$$\Phi_i = \vec{E}_i \cdot \Delta\vec{A}_i$$ flux due to patch i

Net flux through entire surface is:

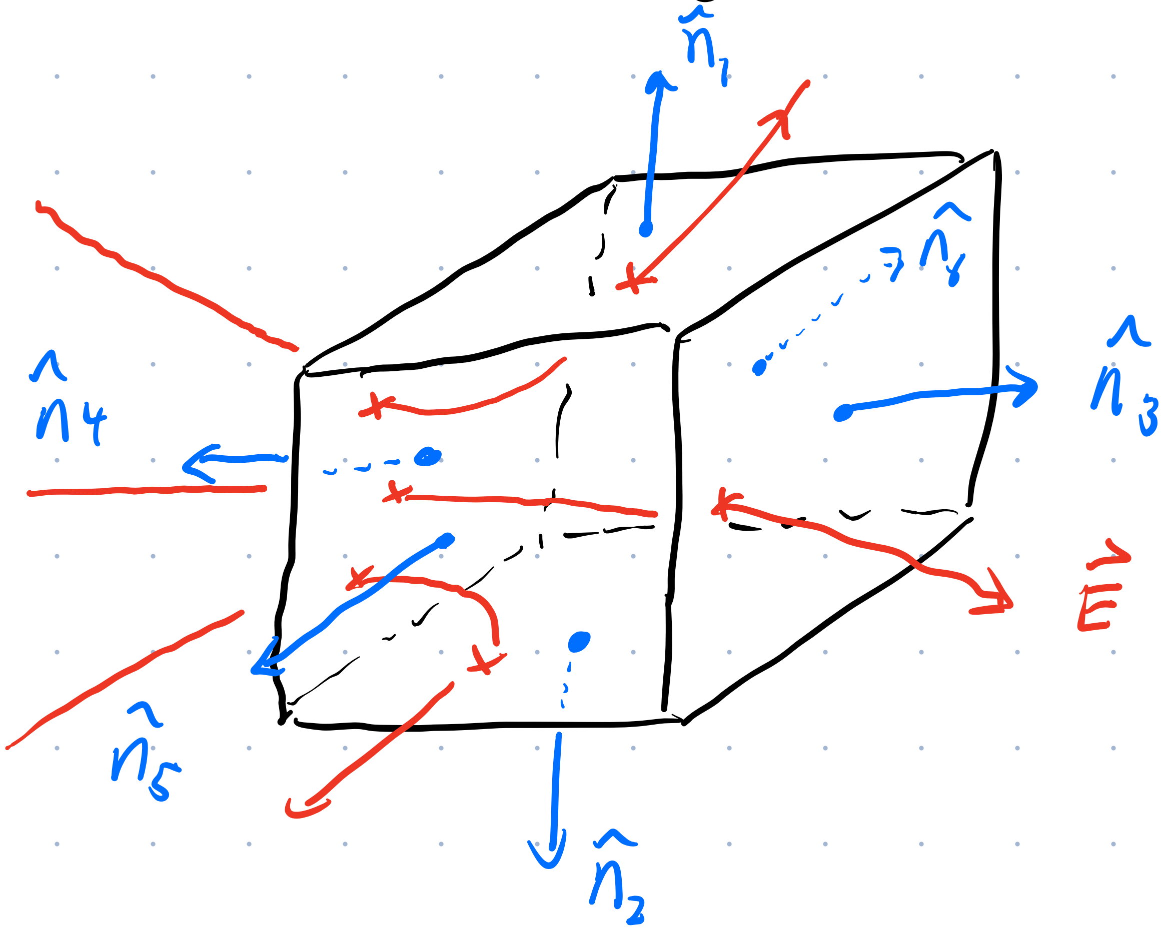
$$\Phi \approx \underbrace{\sum_i \Phi_i}_{\text{sum all contributions from all sub-areas.}} = \sum_i \vec{E}_i \cdot \Delta\vec{A}_i$$

In the limit that $\Delta A \to 0$, the approx. calc. for Φ becomes exact & the sum is expressed as an integral:

$$\Phi = \lim_{\Delta A \to 0} \sum_i \vec{E}_i \cdot \Delta\vec{A}_i = \int \vec{E} \cdot d\vec{A}$$

$$\boxed{\therefore \quad \Phi = \int \vec{E} \cdot d\vec{A}}$$ any surface in any $\vec{E}$.

Let's now consider a closed surface in an arbitrary electric field. (We will require Φ through closed surfaces when using Gauss's Law, next week).



For a closed surface, define our normal vectors $\hat{n}$ such that they point outwards.

When $90 < \theta < 180^\circ$, $\cos\theta < 0$ (negative).
Example $\hat{n}_4$ makes an angle of approx. 180°
$\therefore \cos\theta < 0$

$\Rightarrow$ negative flux when $\vec{E}$ enters a closed surface.

When $0 < \theta < 90^\circ$; $\cos\theta > 0$ (positive)

Example $\hat{n}_3$ makes a small angle with $\vec{E}$

$\Rightarrow$ positive flux when $\vec{E}$ exits a closed surface.