PHYS 121 Jan. 20, 2025

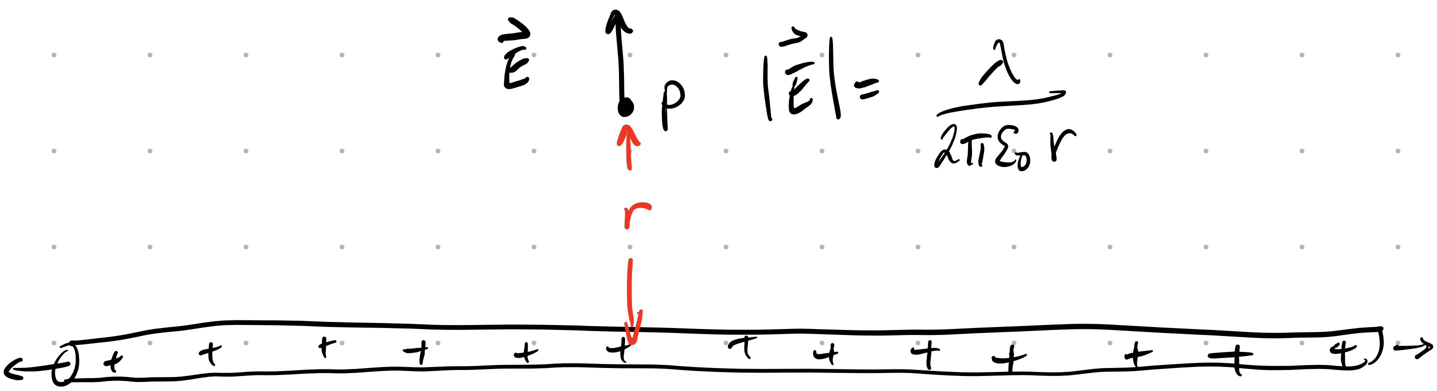
To do:

- complete HW3 on PL by Jan. 24 @ 23:59
- Labs & Tutorials start this week

  There is no pre-lab for the first lab.

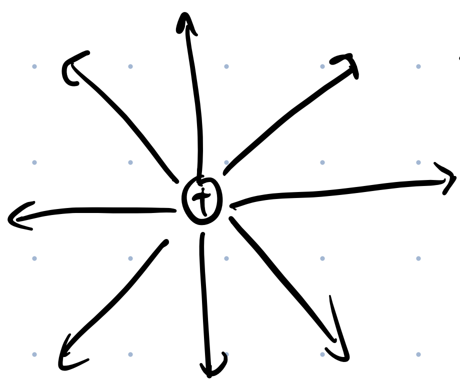
Last Time:

Electric field due to an infinite line of charge with charge per unit length λ.



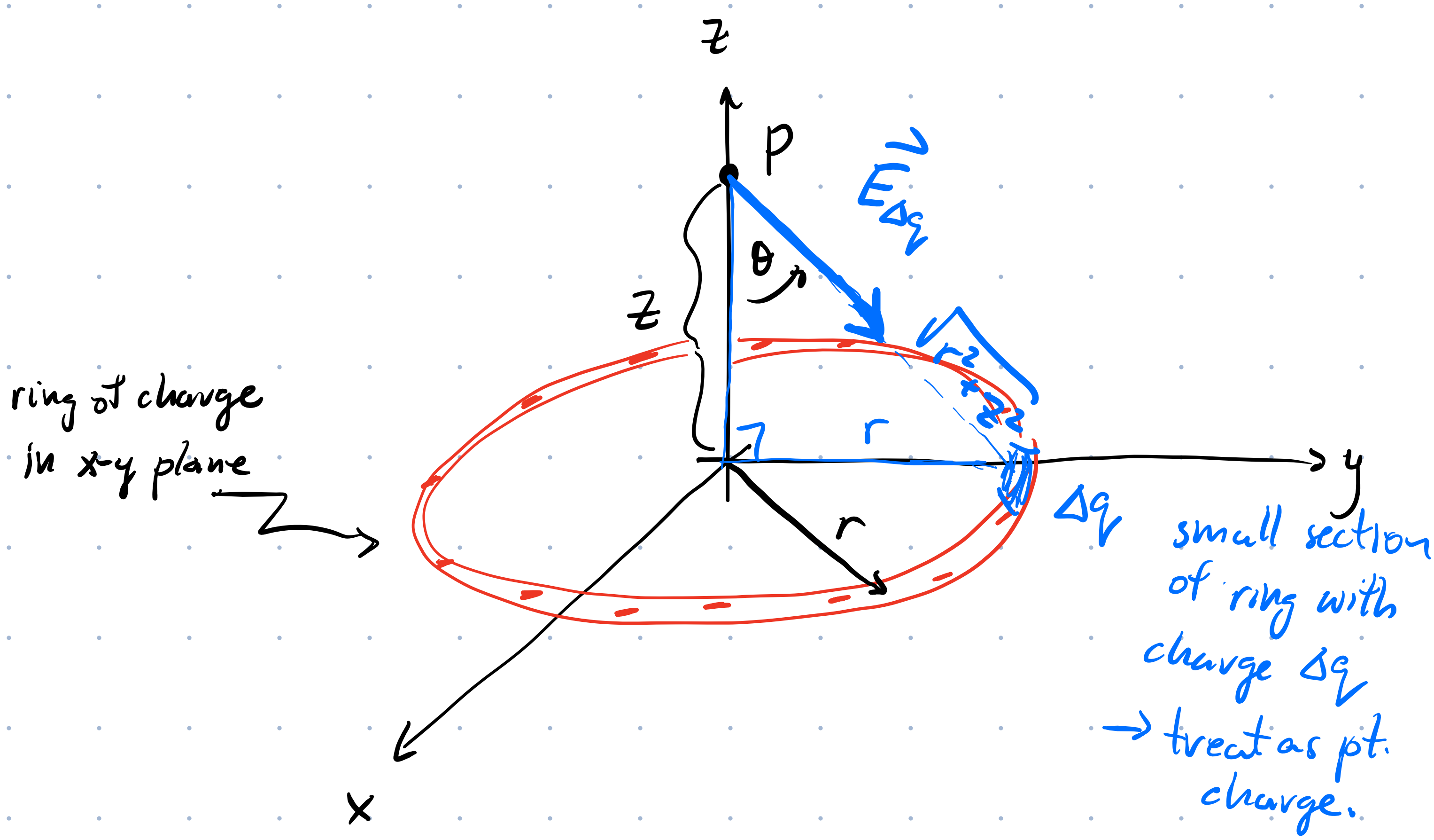
$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 r}$$

End view



$\vec{E}$ points radially away from wire.

Today: $\vec{E}$ due to a ring of charge.



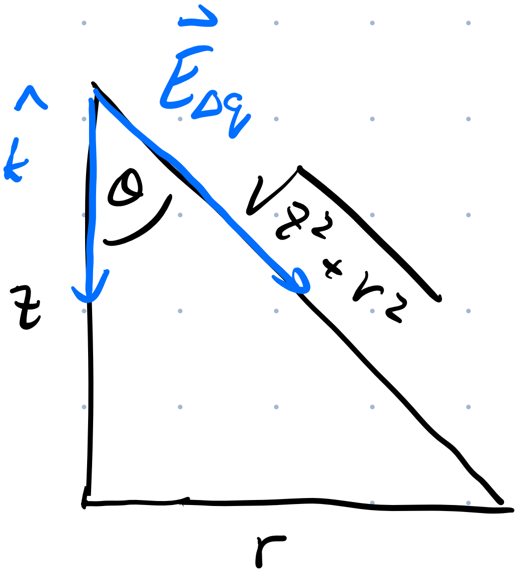
Find $\vec{E}$ @ a point P along the z-axis which passes through the centre of a uniformly-charged ring of radius r. Assume the ring has a negative charge $Q_0 < 0$.

Charge per unit length on ring is

$$\lambda = \frac{Q_0}{2\pi r} \longleftarrow \text{circumference of ring.}$$

By symmetry, the net electric field due to charged ring will pts along -z-axis.

$$\vec{E}_{\Delta q, z} = -E_{\Delta q} \cos\theta \, \hat{k}$$



$$\cos\theta = \frac{z}{\sqrt{z^2+r^2}}$$

$$E_{\Delta q, z} = \underbrace{\left(\frac{k_e \, \Delta q}{z^2+r^2}\right)}_{E_{\Delta q}} \underbrace{\left(\frac{z}{\sqrt{z^2+r^2}}\right)}_{\cos\theta}$$

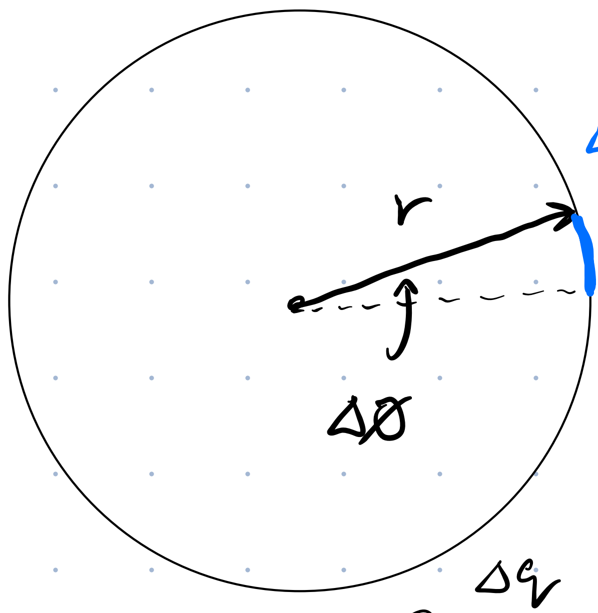
$$= \frac{k_e \, \Delta q \, z}{(z^2+r^2)^{3/2}}$$

vertical component of $\vec{E}$ @ P due to just Δq.

To get the net or total electric field, sum all contribution from all "Δq's" that make up the charged ring.

$$E_{net} = \sum_{\substack{\text{point charges}\\ \text{on ring}}} \left(\frac{k_e \Delta q \, z}{(z^2+r^2)^{3/2}} \right)$$

Top view of ring.



$\Delta q = \lambda \Delta s = \lambda r \Delta \phi$

$\Delta s = r \Delta \phi$

$$\therefore \; E_{net} = \sum_{\text{point charges}} \frac{k_e \overbrace{(\lambda r \Delta \phi)}^{\Delta q} z}{(r^2+z^2)^{3/2}}$$

all of k_e, r, λ, z are const. as we move around ring.

These quantities can be taken outside the sum.

$$E_{net} = \frac{k_e \lambda r z}{(r^2+z^2)^{3/2}} \underbrace{\sum_{\substack{\text{all}\\ \text{point}\\ \text{charges}}} \Delta\phi}_{\substack{\text{all up all } \Delta\phi\text{'s round}\\ \text{the ring} \Rightarrow 2\pi \text{ rad.}}}$$

$$E_{net} = \frac{k_e (2\pi r \lambda) z}{(r^2+z^2)^{3/2}}$$

recall that $\lambda = \dfrac{Q_0}{2\pi r}$

$$\therefore 2\pi r \lambda = Q_0$$

$$\boxed{\vec{E}_{net} = \frac{k_e Q_0 z}{(z^2+r^2)^{3/2}} \hat{k}}$$

$\hat{k}$ is a unit vector in the z-dir'n.

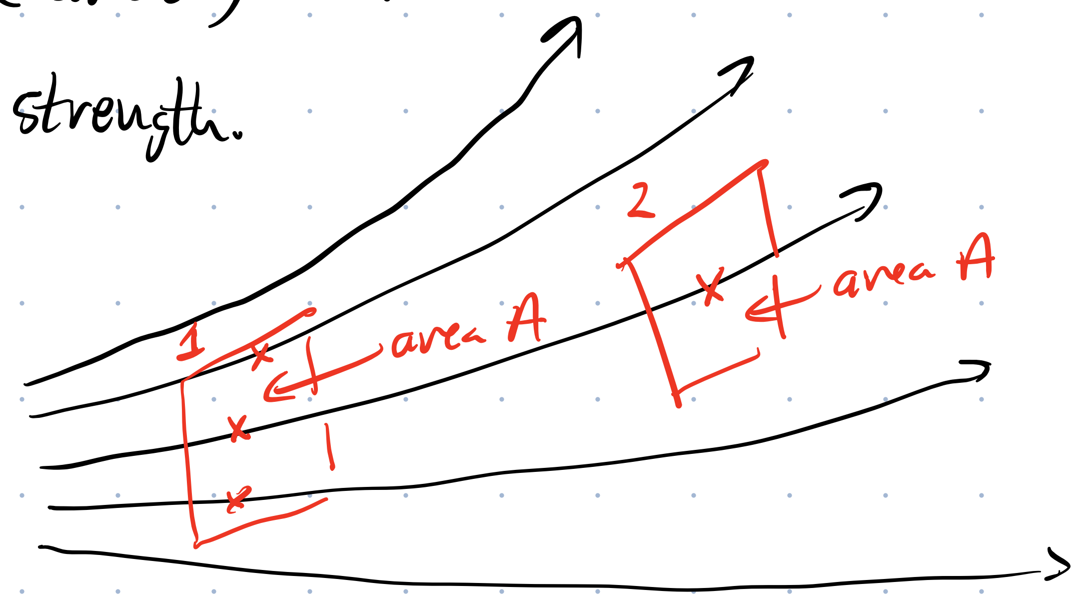
OSUPv2 Chapter 6: Electric Flux

- Why do we care about flux?

  - The electric flux will help us calculate the electric field due to some charge distributions ⇒ Gauss's Law

  - We will see later that changing magnetic flux induces voltages ⇒ Faraday's Law.

Recall that the density of electric field lines $\left(\frac{\#\text{ lines}}{\text{area}}\right)$ is proportional to the electric field strength.



In the example above, three times the no. of $\vec{E}$-field lines through area @ pos. 1 than at pos 2.

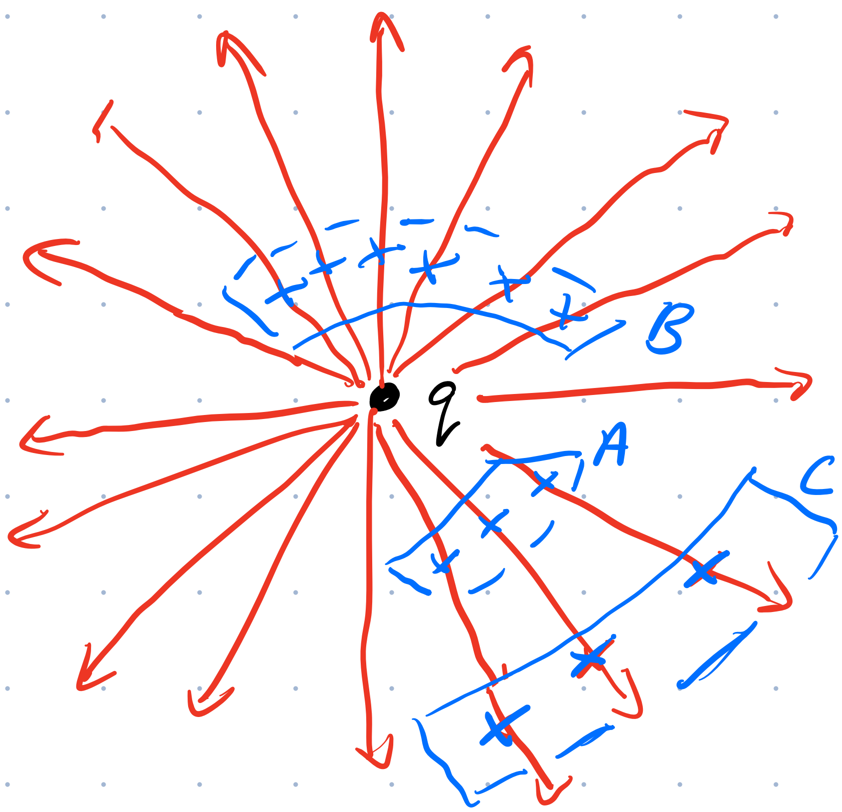
$$\therefore \quad \frac{|\vec{E}_2|}{|\vec{E}_1|} = \frac{1}{3}$$

Greek letter capital Φ (lower case ϕ)

The electric flux Φ is defined to be the electric field strength times area of surface.

$$\text{Since} \quad E \propto \frac{\#\text{lines}}{\text{area}} \qquad \Phi \propto E \cdot \text{area} \propto \#\text{lines}$$

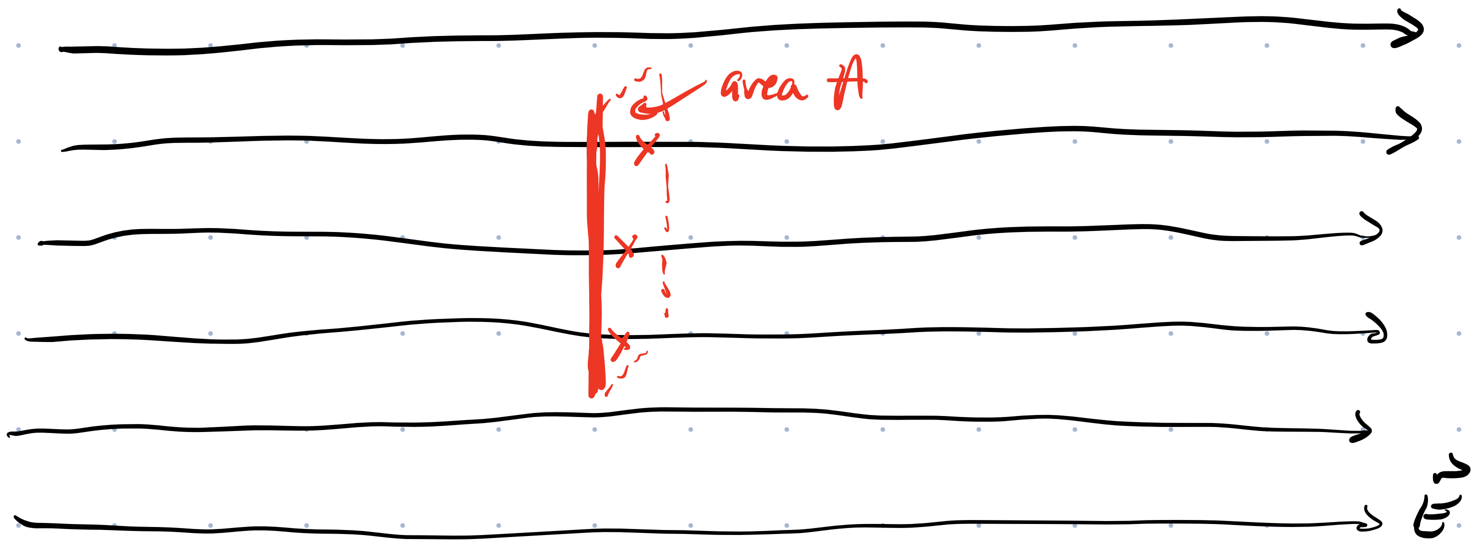
$\vec{E}$-field of pt. Charge.



In the figure above areas of B & C are equal.
A has half the area of B & C.

Electric Field	Electric Flux
prop. to density of lines	prop. to # of lines through a surface
$E_A = E_B$	$\Phi_A = \Phi_C$
$E_C = \frac{E_A}{2} = \frac{E_B}{2}$	$\Phi_B = 2\Phi_A = 2\Phi_C$

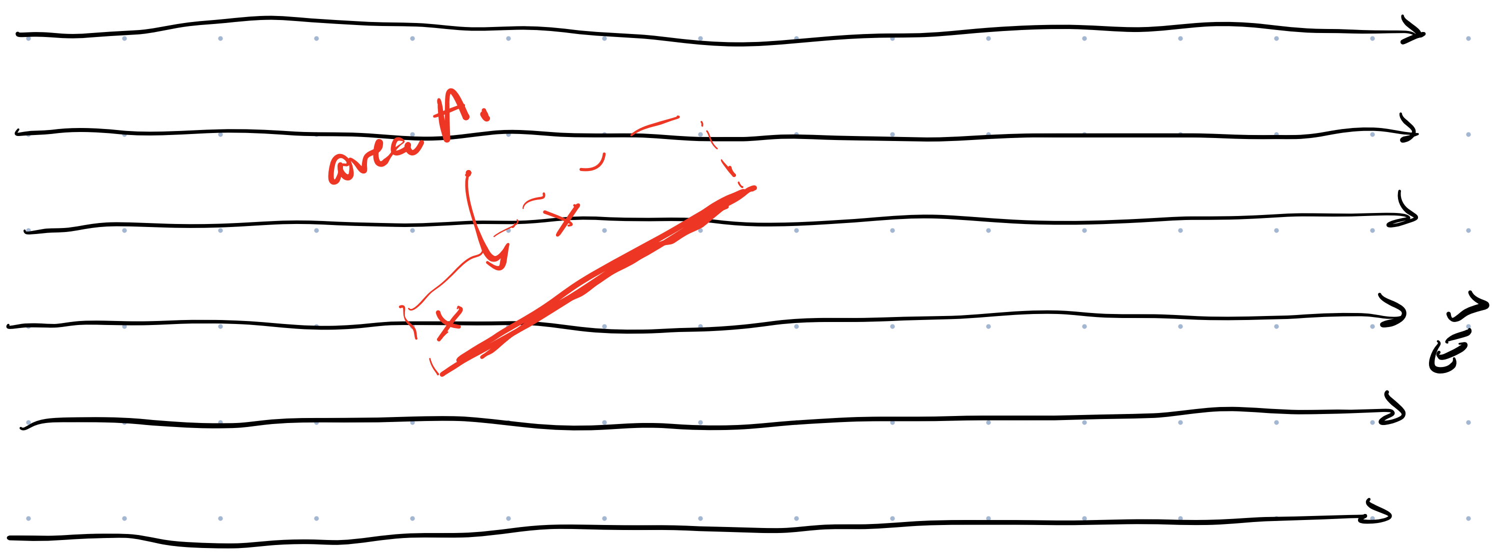
Imagine a uniform electric field & a surface $\perp$ to $\vec{E}$



In this simple case, the electric flux is given by:

$$\Phi = EA$$

Next, imagine the uniform $\vec{E}$ & same surface, except we now tilt the surface relative to dir'n of $\vec{E}$.



When surface is not $\perp$ to $\vec{E}$, flux is reduced because fewer field lines pass through it.