PHYS 121

Jan. 17, 2025

To do:

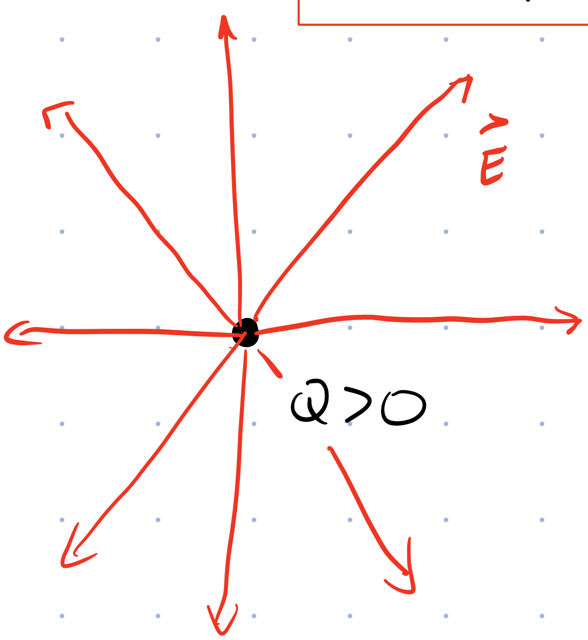
- complete HW2 on PL by $\boxed{\text{today}}$ @ 23:59
- Labs & Tutorials start the week of Jan. 20.

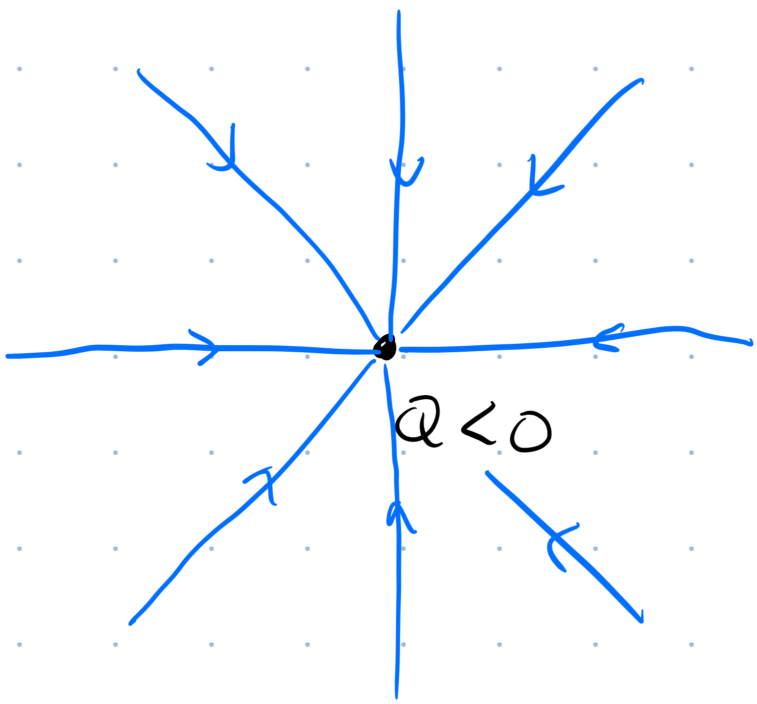
  There is <u>no</u> pre-lab for the first lab.

<u>Last Time:</u>

Electric field due to a point charge.

$$\boxed{\vec{E} = \frac{k_e Q}{r^2}\hat{r}} \quad (1)$$

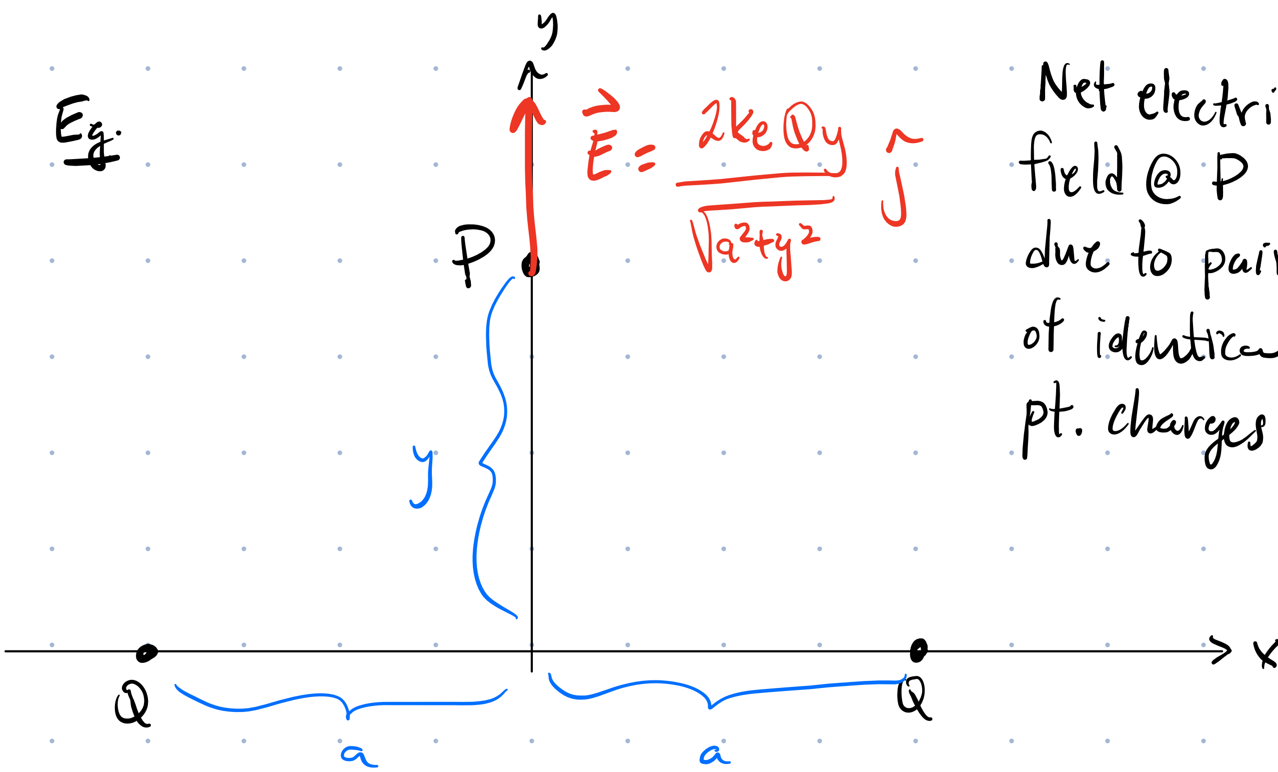




Force on point charge q_0 place at a pt. P where the electric field is given by $\vec{E}$:

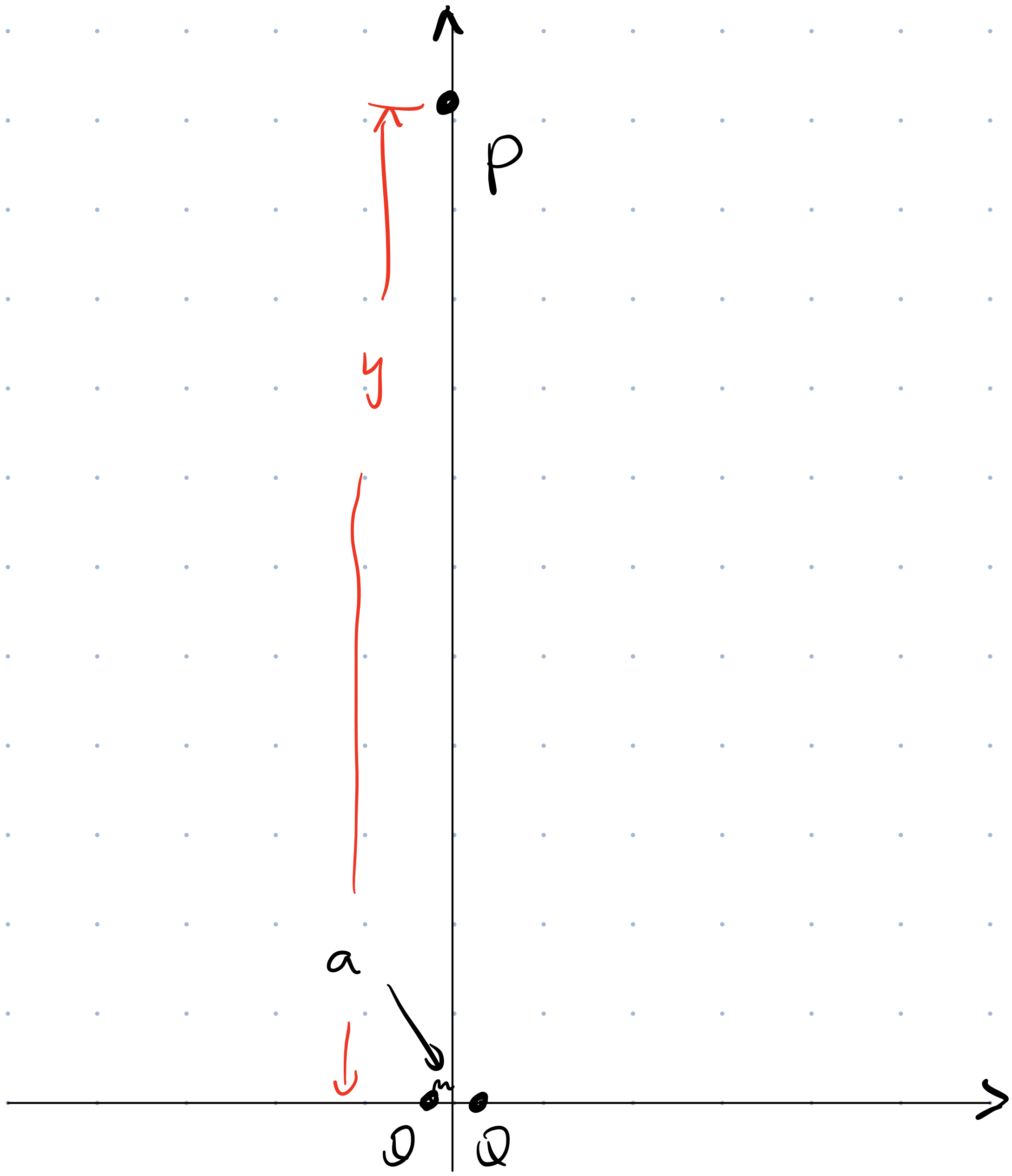
$$\boxed{\vec{F} = q_0 \vec{E}}$$

Eg.



Net electric field @ P due to pair of identical pt. charges Q

Consider what happens when we make $y \ll a$ i.e. P is very far from the two pt. charges.



From the position of P, it looks like we have a total charge of 2Q at the origin.

Expect electric field @ P to look like that of a pt. charge of 2Q a distance y away:

$$\boxed{E \approx \frac{k_e(2Q)}{y^2}}$$ 2

Expect Eq. ① to become ② when $y \gg a$.

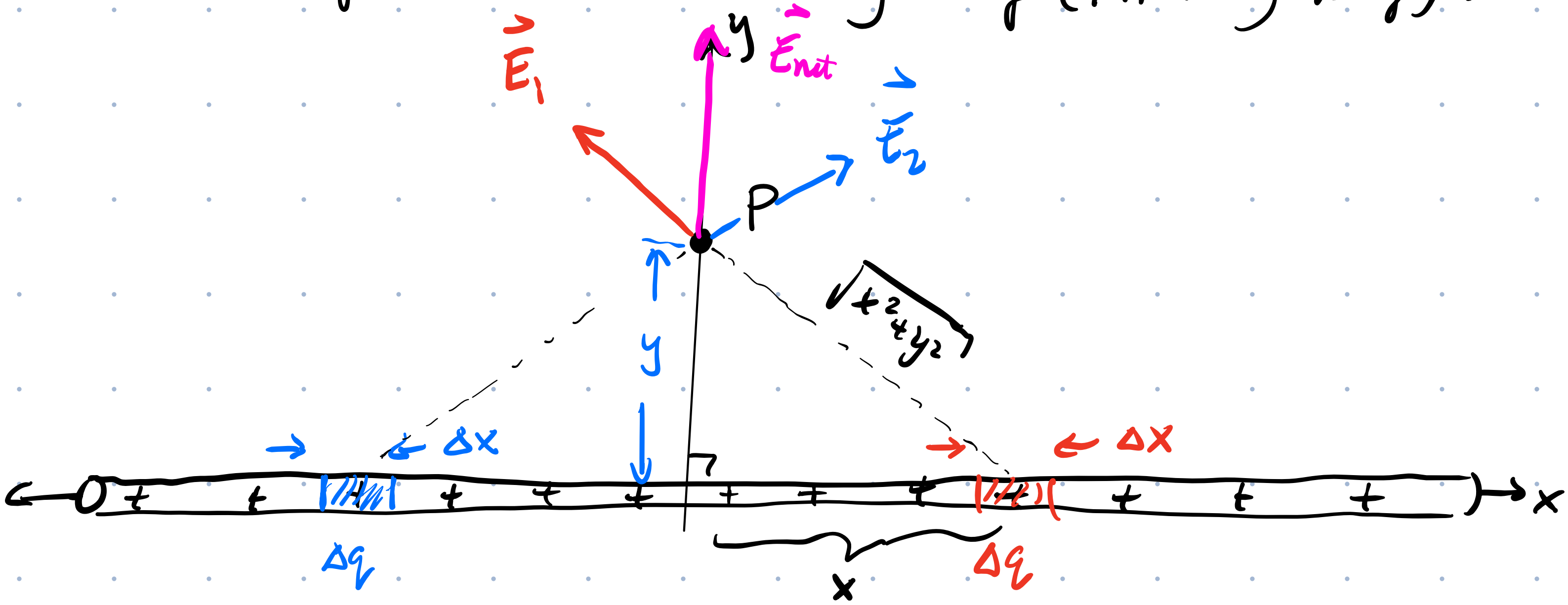
Eq. ① $$E = \frac{k_e\, 2Q y}{\boxed{(y^2+a^2)^{3/2}}}$$

If $y \gg a$, then $y^2 + a^2 \simeq y^2$

$$(y^2+a^2)^{3/2} \approx (y^2)^{3/2} \approx y^3$$

$$E \approx \frac{k_e\, 2Q\cancel{y}}{y^{\cancel{3}\,2}} \approx \frac{k_e(2Q)}{y^2}$$ Eq. ②, as expected!

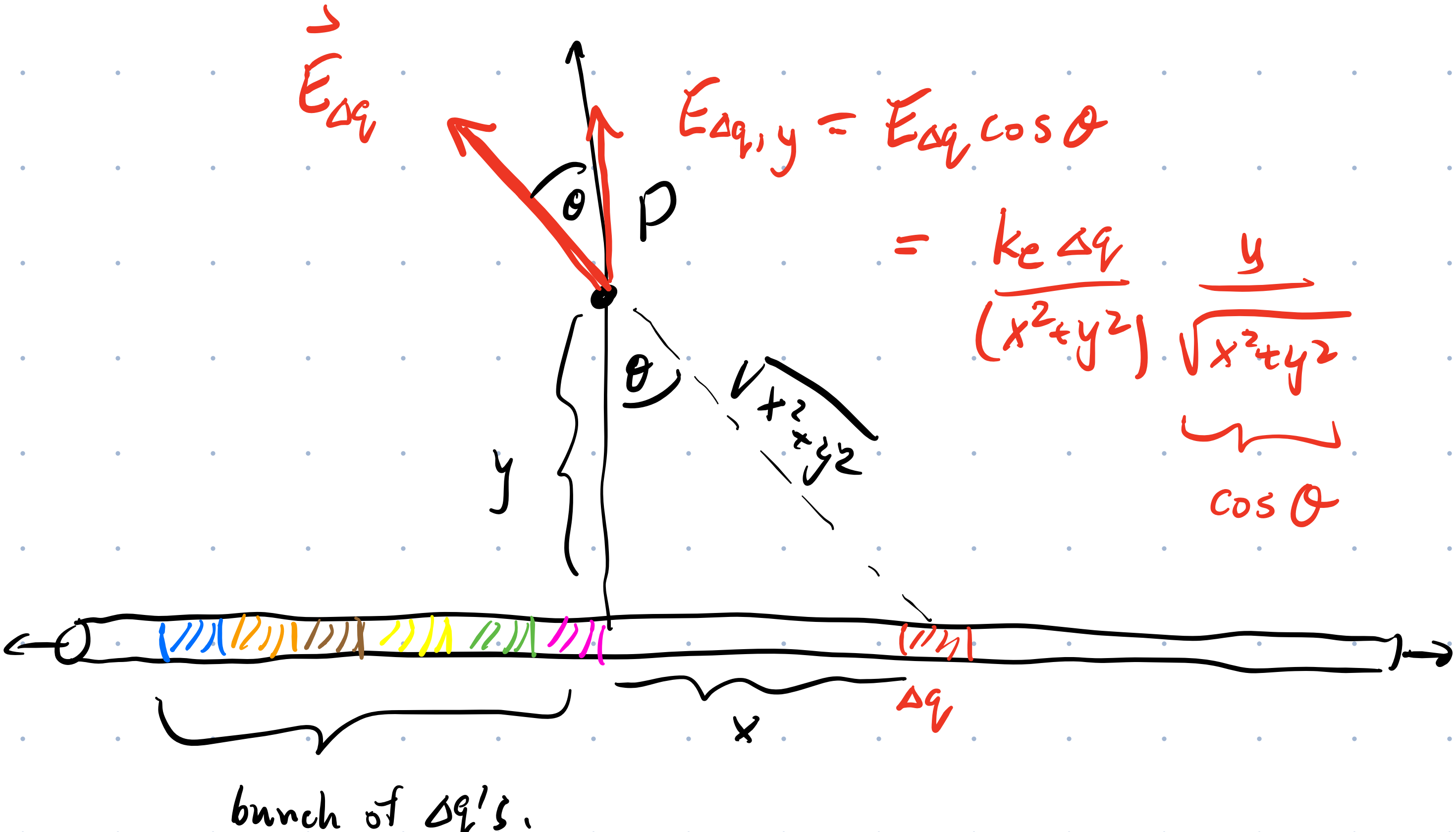
Calculate the electric field due to a uniformly-charged rod that is very long (infinitely long).



Select a section of rod of length Δx (small). That section of rod has charge Δq which we treat as a pt. charge. Δq contributes an electric field $\vec{E}_1$ @ pt. P.

For every Δq to the right of y-axis (red), there is a corresponding Δq placed symmetrically on the left (blue). The pairs of charges create $\vec{E}$ @ P with x-components the cancel and y-components that add.

Therefore, by symmetry, the net electric due to the uniformly-charged rod points perpendicularly away from the rod.



$$E_{\Delta q, y} = E_{\Delta q} \cos\theta$$

$$= \frac{k_e \Delta q}{(x^2+y^2)} \underbrace{\frac{y}{\sqrt{x^2+y^2}}}_{\cos\theta}$$

$$E_{\Delta q, y} = \frac{k_e \, \Delta q \, y}{(x^2+y^2)^{3/2}}$$

The net electric field due to entire rod is found by adding up contributions from all Δq's that make up the charged rod.

$$E_{net} = \sum_{\substack{i \\ \text{all } \Delta q\text{'s}}} \frac{k_e \boxed{\Delta q} \, y}{(x_i^2 + y^2)^{3/2}}$$

Assume that a section of rod of length L has total charge Q. We can then define a linear charge density (charge per unit length) of

Greek letter lambda $\rightarrow \lambda = \frac{Q}{L}$

$\therefore$ In a section of length Δx, we have a charge of

$$\boxed{\Delta q = \lambda \, \Delta x}$$

$$\therefore \; E_{net} \approx \sum_{\substack{i \\ \text{all } \Delta q\text{'s}}} \frac{k_e \, \lambda \, \Delta x \, y}{(x_i^2 + y^2)^{3/2}}$$

In the limit that $\Delta x \to 0$, the Δq's become more pt.-charge-like & our calculation of $\vec{E}_{net}$ improves.

$$E_{net} = \lim_{\Delta x \to 0} \sum_i \frac{k_e \lambda y}{(x_i^2 + y^2)^{3/2}} \Delta x$$

Reimann sum which we can express as an integral.

$$= \int \frac{k_e \lambda y}{(x^2+y^2)^{3/2}} dx$$

The result is:

$$\boxed{E_{net} = \frac{2 k_e \lambda}{y}}$$

using $k_e = \frac{1}{4\pi\varepsilon_0}$

$$\boxed{E_{net} = \frac{\lambda}{2\pi\varepsilon_0 y}}$$

electric field due to a uniformly-charged rod. It points $\perp$ to rod.