PHYS 121 Jan. 13, 2025

To do:

- complete HW1 on PL by $\boxed{\text{today}}$ @ 23:59 (link in Canvas)
- complete HW2 on PL by Jan. 17 @ 23:59
- Labs & Tutorials start the week of Jan. 20.

Last Time:

Force between a pair of pt. charges

$$\boxed{|\vec{F}_{12}| = |\vec{F}_{21}| = k_e \frac{|q_1||q_2|}{r^2}}$$

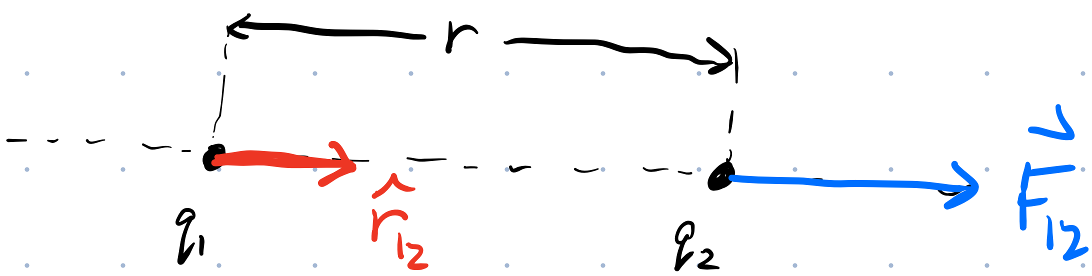
Coulomb's Law

attractive for opposite charges
repulsive for like charges

$k_e = 8.99 \times 10^9 \frac{Nm^2}{C^2}$

Today: Start w/ vector form of Coulomb's Law.

Goal: Include information about dir'n of force in Coulomb's law.



Define a unit vector $\hat{r}_{12}$

- a vector of length 1 ; $|\hat{r}_{12}| = 1$.
- $\hat{r}_{12}$ points from q_1 to q_2

Consider the vector eq'n

$$\vec{F}_{12} = \frac{k_e q_1 q_2}{r^2} \hat{r}_{12}$$

- Like charges (either q_1 & q_2 both pos. or both neg.)

$q_1 \cdot q_2 > 0$

The product $q_1 q_2$ is always pos. in this case.

In this case $\vec{F}_{12}$ force on on q_2 due to q_1 is in the <u>same</u> dir'n as $\hat{r}_{12}$. ✓

- Opposite charges (one pos. & one neg.)

$q_1 q_2 < 0$

i.e. the product $q_1 q_2$ is <u>negative</u>

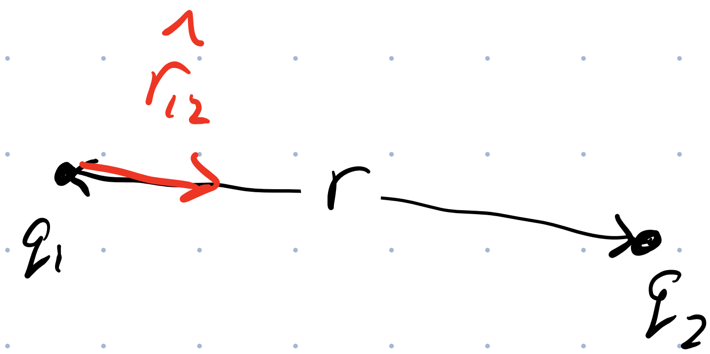
$\therefore \vec{F}_{12}$ is going to be in the opposite dir'n of $\hat{r}_{12}$. This makes sense since, in this case, q_2 is attracted to q_1. ✓

Summary: Coulomb's Law (vector version)

Force on q_2 due to q_1 is:

$$\boxed{\vec{F}_{12} = \frac{k_e q_1 q_2}{r^2}\hat{r}_{12}}$$

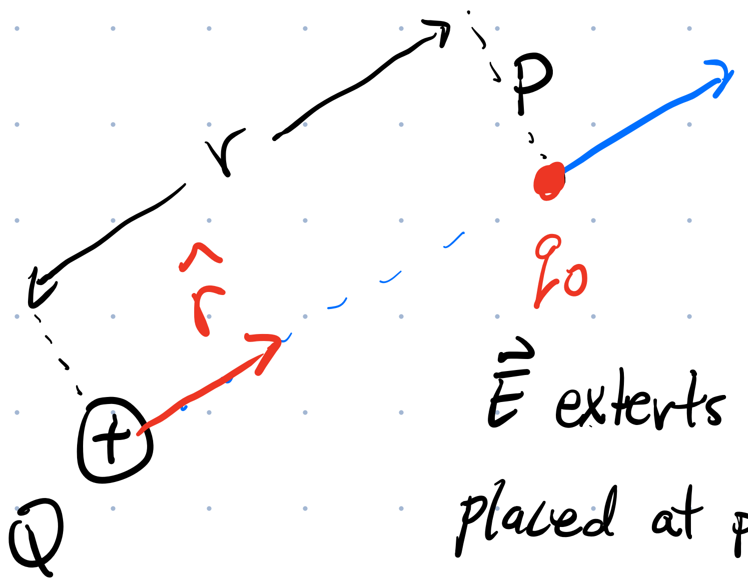
Contains the dir'n & mag. of the force.



5.4 in OSUPv2: The Electric Field

Use the symbol $\vec{E}$ for electric field.

A charged object generates an electric field that exerts forces on other nearby charges. The strength of the force is prop. to the strength of the electric field.



$\vec{E}$ Electric field @ P due to Q.

$\vec{E}$ exterts a force on q_0 placed at pt. P.

By definition, the force on q_0 is $\vec{F} = q_0 \vec{E}$. ①

From Coulomb's law, know that force on q_0 is given by:

$$\vec{F} = \frac{k_e q_0 Q}{r^2} \hat{r} \quad ②$$

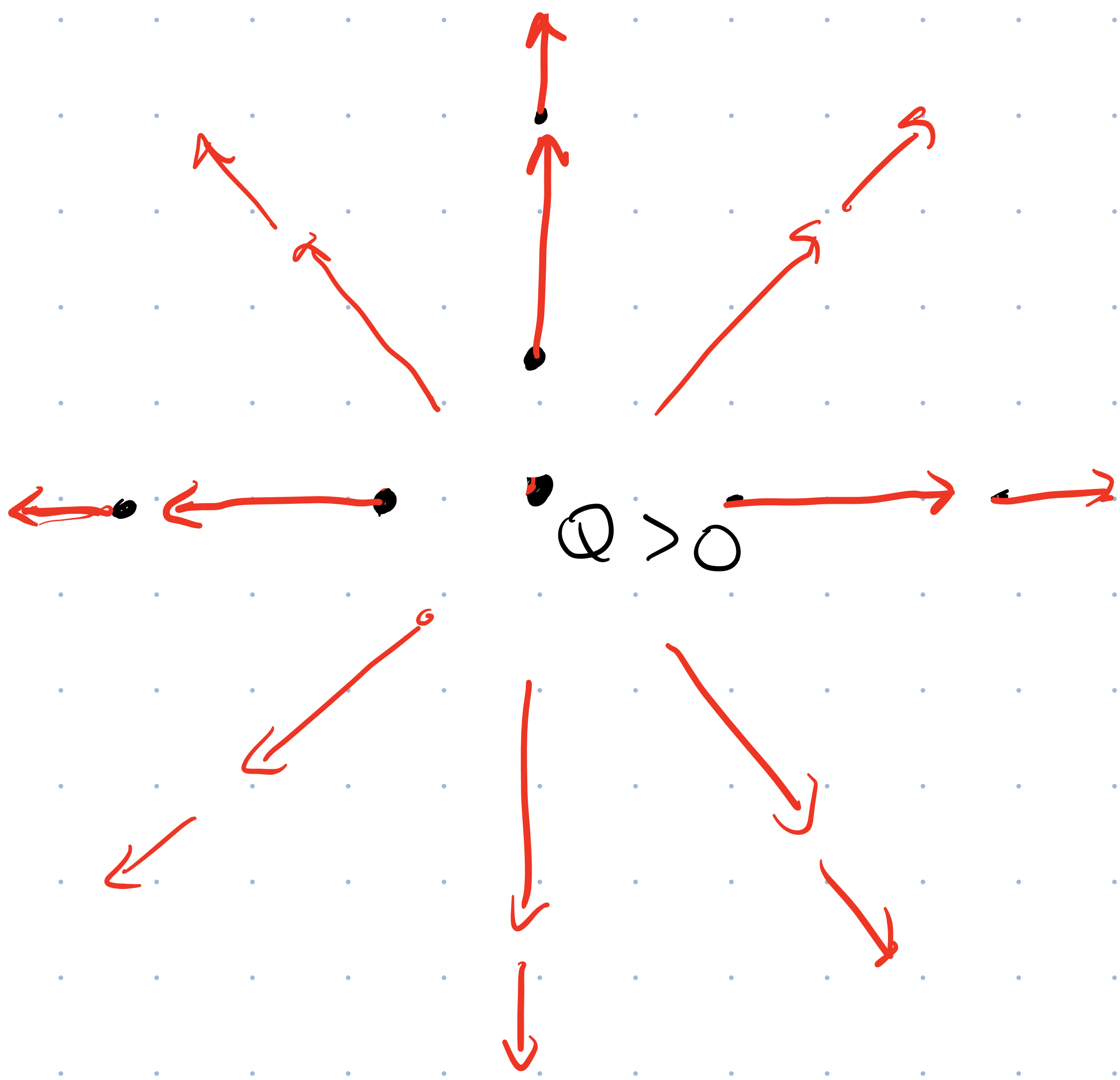
① & ② are two eq'ns for the same force & must be equal.

$$\cancel{q_0} \vec{E} = \frac{k_e \cancel{q_0} Q}{r^2} \hat{r}$$

$$\boxed{\therefore\ \vec{E} = \frac{k_e Q}{r^2}\hat{r}}$$

Electric field due to Q.

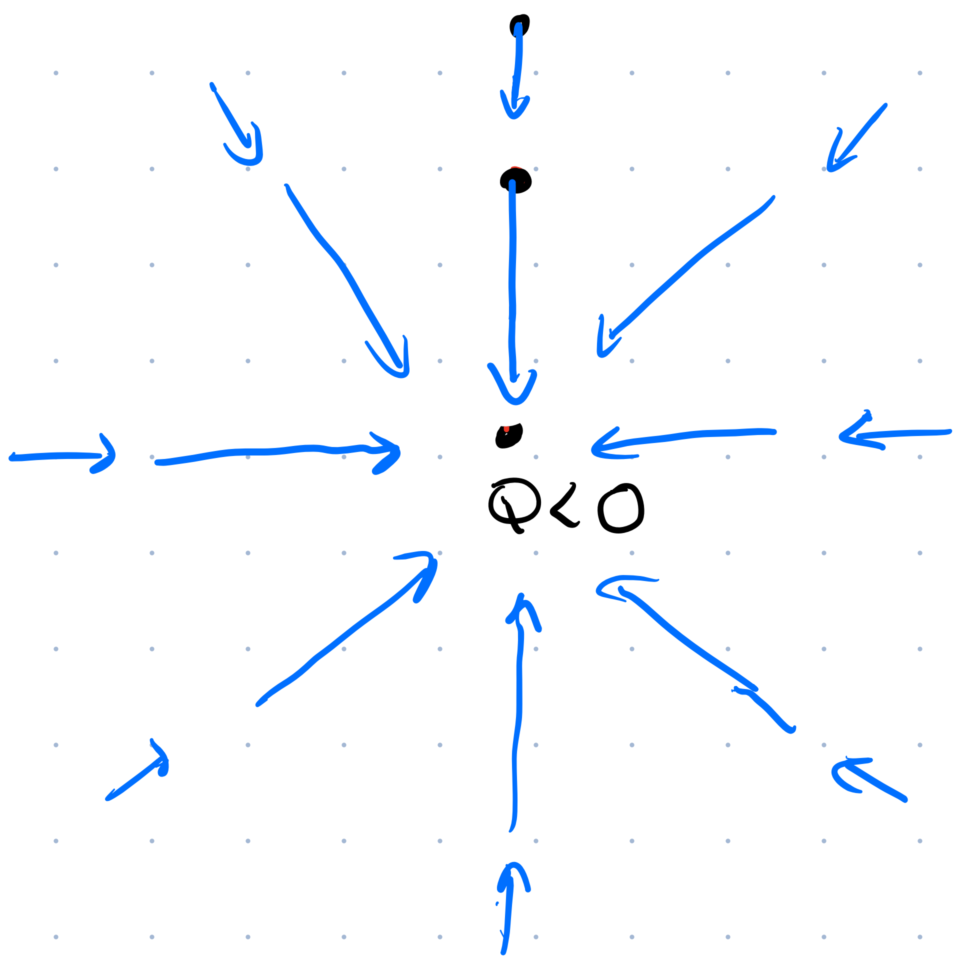
Try to map out the electric field due to a positive pt. charge.



For a positive Q, $\vec{E}$ points radially outwards. $\vec{E}$ decreases in mag. as we move further away from Q.

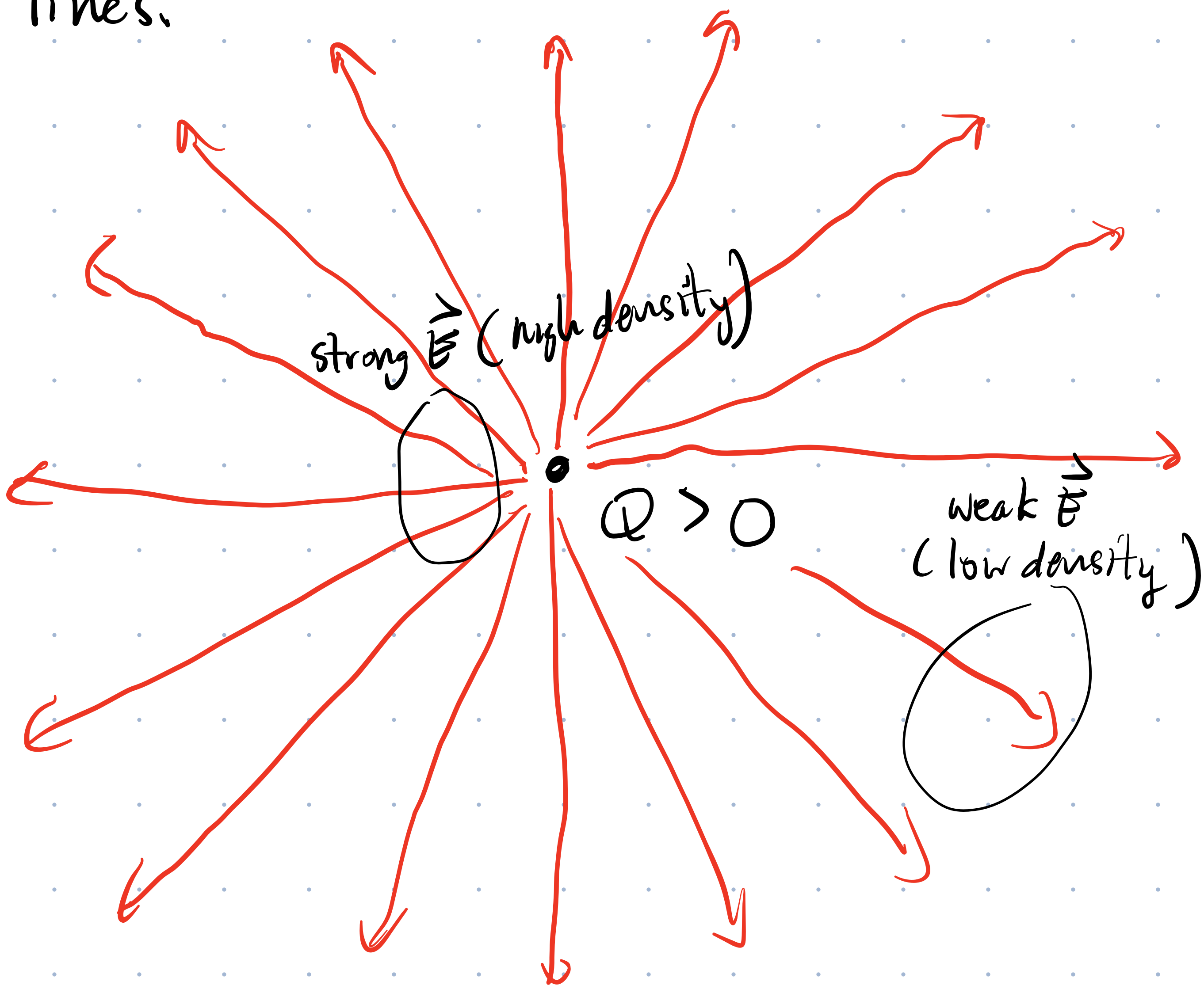
Map out $\vec{E}$ due to a negative Q.

$$\vec{E} = \frac{k_e Q}{r^2} \hat{r}$$



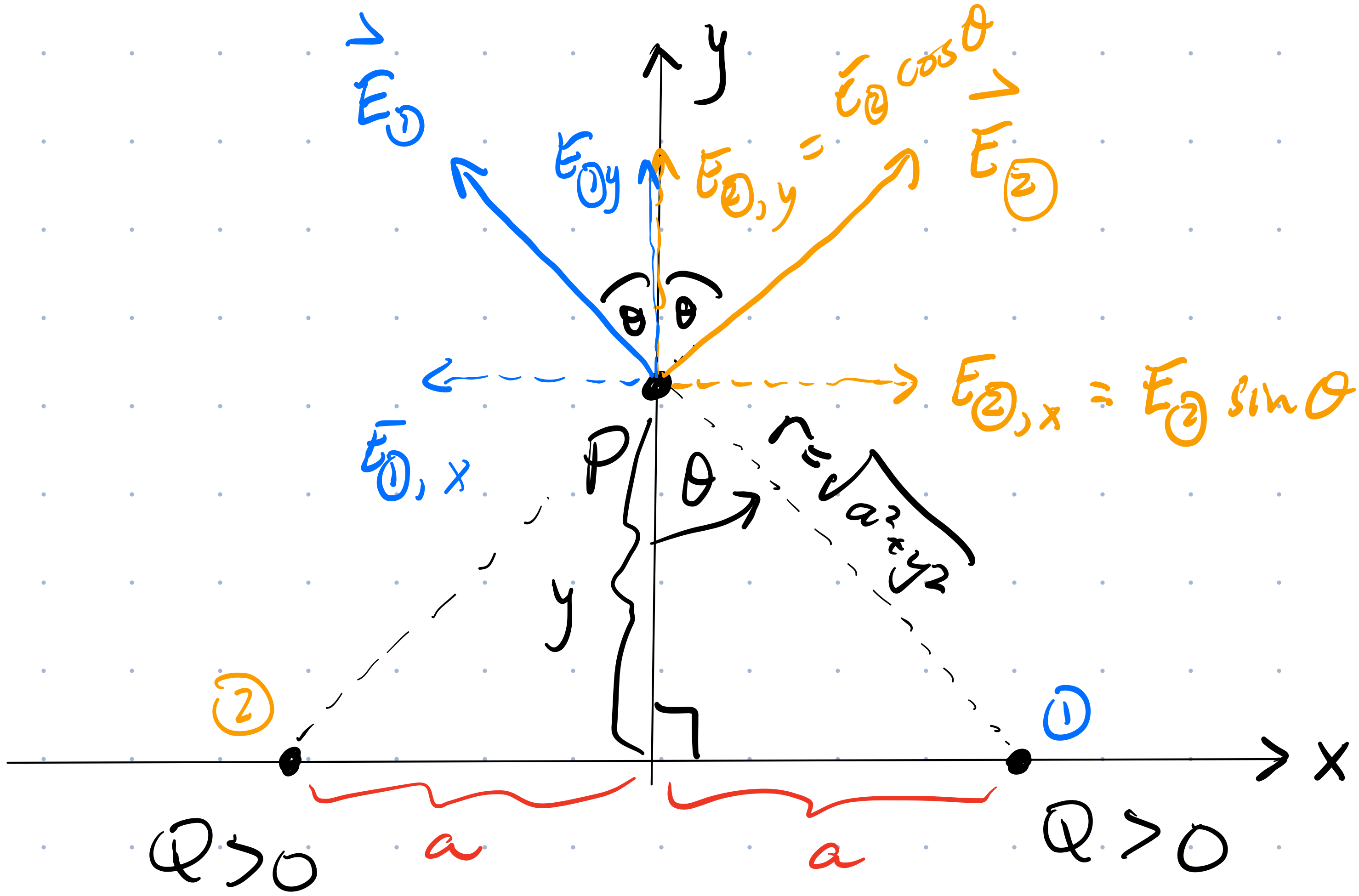
For negative pt. charges, $\vec{E}$ points radially inwards.

Usually draw $\vec{E}$-field lines as continuous lines.



In this picture, the <u>density</u> of $\vec{E}$-field lines tells the strength of the electric field. Where the lines are closely space $\vec{E}$ is strong.

Example: Find the net electric field at pt. P.



Two pt. charges Q are identical & positive.

- Find $\vec{E}$ due to each pt charge individually
- Break the two $\vec{E}$-fields into x & y components
- Add the components to find the net electric

field at P.

$$|\vec{E}_{②}| = |\vec{E}_{①}| = \frac{k_e Q}{a^2+y^2}$$

x-components.

$$|E_{①,x}| = |E_{②,x}| = \frac{k_e Q}{a^2+y^2} \sin\theta$$

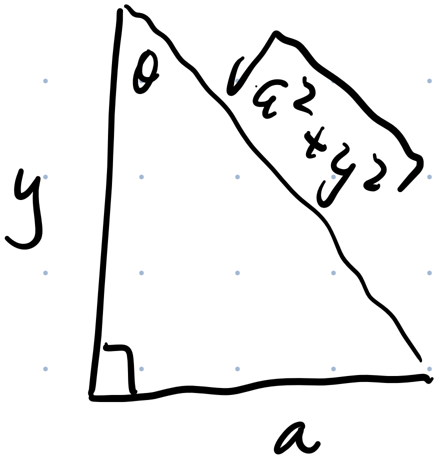
y-components

$$|E_{①,y}| = |E_{②,y}| = \frac{k_e Q}{a^2+y^2} \cos\theta$$

x-components cancel b/c they're opp. dir'ns.

y-components add.

$$E_{y,net} = 2\frac{k_e Q}{a^2+y^2} \cos\theta$$



$$\cos\theta = \frac{y}{\sqrt{a^2+y^2}}$$

$$\therefore E_{y\,net} = \frac{2k_eQ}{a^2+y^2}\,\frac{y}{\sqrt{a^2+y^2}}$$

$$\boxed{\vec{E}_{net} = \frac{2k_eQy}{(a^2+y^2)^{3/2}}\,\hat{y}}$$