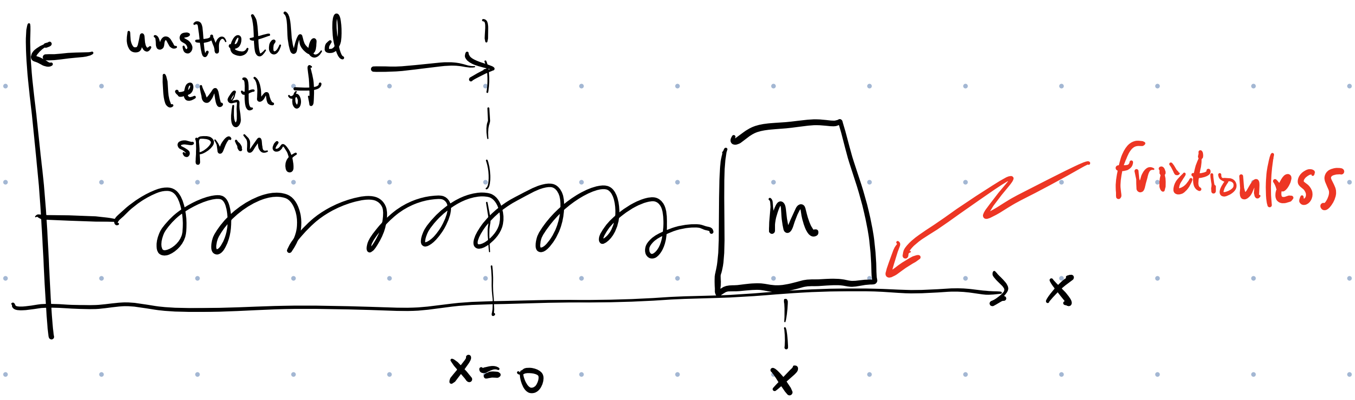
PHYS 121

Jan. 8, 2025

To do:
- complete survey by Jan. 13 @ 23:59 (link in Canvas)
- complete HW1 on PL by Jan. 15 @ 23:59 (link in Canvas)
- complete HW2 on PL by Jan. 17 @ 23:59

Today: Some review of PHYS 111
- Eq'ns of motion
- Free-body diagrams
- Mass on a spring
- Rotational motion
- Pendulum (Labs #1 & 2.)

Mass on Spring



Spring force: Hooke's Law $\vec{F}_s = -k\vec{x}$

spring const.

Newton's 2nd Law in x-dir'n

$$F_x = ma_x = -kx \quad \text{(Eq'n of motion)}$$

Can solve for the position of mass x as a fcn of time.

$$ma_x = -kx \qquad a_x = \frac{dv}{dt}$$

$$m\frac{dv}{dt} = -kx \qquad v = \frac{dx}{dt}$$

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$\therefore \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

$$\boxed{m\frac{d^2x}{dt^2} = -kx} \quad (1)$$

To find x(t), require a fcn of t that, after 2 derivatives, returns the original fcn w/ a minus sign.

Try a sol'n of the form $x = A\cos(\omega t)$ angular freq. (period)

amplitude / initial displacement

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t)$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 \underbrace{A\cos(\omega t)}_{x}$$

$$\boxed{\therefore \frac{d^2x}{dt^2} = -\omega^2 x} \quad (2)$$

Sub (2) into (1):

$$m(\cancel{-}\omega^2 \cancel{x}) = \cancel{-} k \cancel{x}$$

solve for $\boxed{\omega = \sqrt{\frac{k}{m}}}$ angular freq. of mass on a spring.

Recall that angular freq $\omega = 2\pi f$ where f is the no. of cycles per second.

Period $T = \frac{1}{f}$ is the time require to complete one cycle or one osc. of the motion.

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

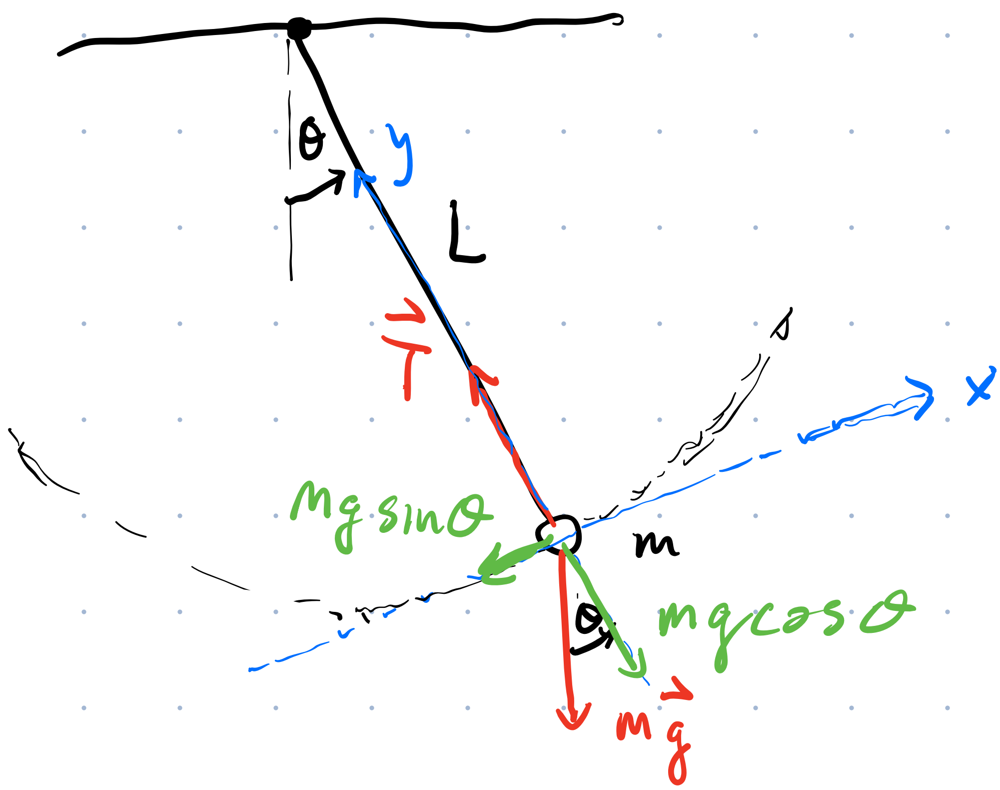
$\therefore$ solving for the period gives:

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Notice that the period of the motion is <u>independent</u> of the amplitude.

Pendulum : Try to relate the prob. of an osc. pendulum to the mass on a spring.

Free-body diagram.



Decompose our forces into x- & y-components. The position of the mass of the pendulum is specified by the angular variable θ.

2nd Law in y-dir'n:

$$F_y = ma_y = 0 = T - mg\cos\theta$$

$$\boxed{\therefore T = mg\cos\theta}$$

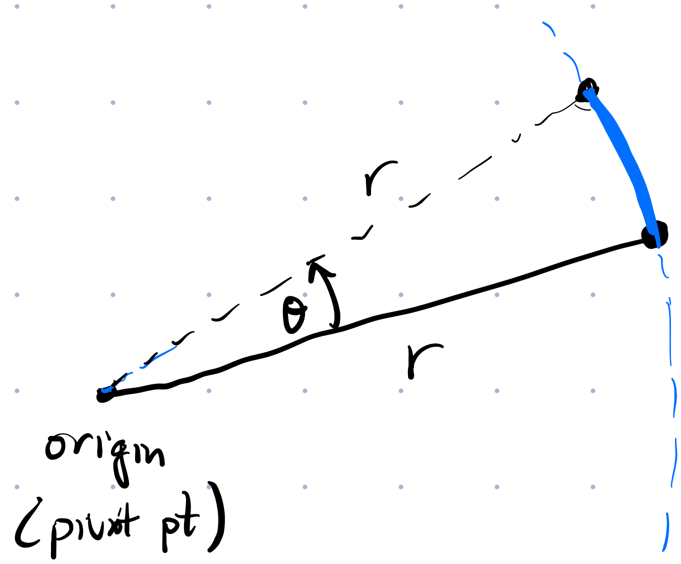
2nd Law in x-dir'n:

$$F_x = ma_x = -mg\sin\theta$$

$$\boxed{\therefore a_x = -g\sin\theta} \quad (3)$$

Eq. (3) is not quite the same as mass on a spring. Let's see if we can make some manipulations to make them more similar.

Aside: Rotational motion.



length of arc

$s = r\theta$

$$v = \frac{ds}{dt}$$

$$= \frac{d}{dt}(r\theta)$$

const. for circular motion.

$$v = r\frac{d\theta}{dt}$$

Also know $a = \frac{dv}{dt} = \frac{d}{dt}\left(r\frac{d\theta}{dt}\right)$

$$\boxed{a = r\frac{d^2\theta}{dt^2}}$$

Back to pendulum. Here, $r = L$

$$\therefore a_x = L\frac{d^2\theta}{dt^2}$$

$$L\frac{d^2\theta}{dt^2} = -g\sin\theta$$ Pendulum.

or $$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

compare to mass on a spring.

$$\boxed{\frac{d^2x}{dt^2} = -\frac{k}{m}x}$$

These are similar, but not identical.

The pendulum has $\sin\theta$ instead of θ...