Introduction to Decision Trees

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A decision tree can be used as a model for a sequential decision problems under uncertainty. A decision tree describes graphically the decisions to be made, the events that may occur, and the outcomes associated with combinations of decisions and events. Probabilities are assigned to the events, and values are determined for each outcome. A major goal of the analysis is to determine the best decisions.

14.1 DECISION TREE STRUCTURE

Decision tree models include such concepts as nodes, branches, terminal values, strategy, payoff distribution, certain equivalent, and the rollback method. The following problem illustrates the basic concepts.

DriveTek Problem, Part A

DriveTek Research Institute discovers that a computer company wants a new storage device for a proposed new computer system. Since the computer company does not have research people available to develop the new storage device, it will subcontract the development to an independent research firm. The computer company has offered a fixed fee for the best proposal for developing the new storage device. The contract will go to the firm with the best technical plan and the highest reputation for technical competence.

DriveTek Research Institute wants to enter the competition. Management estimates a moderate cost for preparing a proposal, but they are concerned that they may not win the contract.

If DriveTek decides to prepare a proposal, and if they win the contract, their engineers are not sure about how they will develop the storage device. They are considering three alternative approaches. The first approach is a very expensive mechanical method, and the engineers are certain they can develop a successful model with this approach. A second approach involves electronic components. The engineers think that the electronic approach is a relatively inexpensive method for developing a model of the storage device, but they are not sure that the results will be satisfactory for satisfying the contract. A third inexpensive approach uses magnetic components. This magnetic method costs more than the electronic method, and the engineers think that it has a higher chance of success.

DriveTek Research can work on only one approach at a time and has time to try only two approaches. If it tries either the magnetic or electronic method and the attempt fails, the second choice must be the mechanical method to guarantee a successful model.

The management of DriveTek Research needs help in incorporating this information into a decision to proceed or not.

Nodes and Branches

Decision trees have three kinds of nodes and two kinds of branches. A decision node is a point where a choice must be made; it is shown as a square. The branches extending from a decision node are decision branches, each branch representing one of the possible alternatives or courses of action available at that point. The set of alternatives must be mutually exclusive (if one is chosen, the others cannot be chosen) and collectively exhaustive (all possible alternatives must be included in the set).

There are two major decisions in the DriveTek problem. First, the company must decide whether or not to prepare a proposal. Second, if it prepares a proposal and is awarded the contract, it must decide which of the three approaches to try to satisfy the contract.

An event node is a point where uncertainty is resolved (a point where the decision maker learns about the occurrence of an event). An event node, sometimes called a "chance node," is shown as a circle. The event set consists of the event branches extending from an event node, each branch representing one of the possible events that may occur at that point. The set of events must be mutually exclusive (if one occurs, the others cannot occur) and collectively exhaustive (all possible events must be included in the set). Each event is assigned a subjective probability; the sum of probabilities for the events in a set must equal one.

The three sources of uncertainty in the DriveTek problem are: whether it is awarded the contract or not, whether the electronic approach succeeds or fails, and whether the magnetic approach succeeds or fails.

In general, decision nodes and branches represent the controllable factors in a decision problem; event nodes and branches represent uncontrollable factors.

Decision nodes and event nodes are arranged in order of subjective chronology. For example, the position of an event node corresponds to the time when the decision maker learns the outcome of the event (not necessarily when the event occurs).

The third kind of node is a terminal node, representing the final result of a combination of decisions and events. Terminal nodes are the endpoints of a decision tree, shown as the end of a branch on hand-drawn diagrams and as a triangle or vertical bar on computer-generated diagrams.

The following table shows the three kinds of nodes and two kinds of branches used to represent a decision tree.

Figure 14.1 Nodes and Symbols

| Type of Node	Written Symbol	Computer Symbol	Node Successor
Decision	square	square	decision branches
Event	circle	circle	event branches
Terminal	endpoint	triangle or bar	terminal value

In the DriveTek problem, the first portion of the decision tree is shown in Figure 14.2.

Figure 14.2 DriveTek Initial Decision and Event
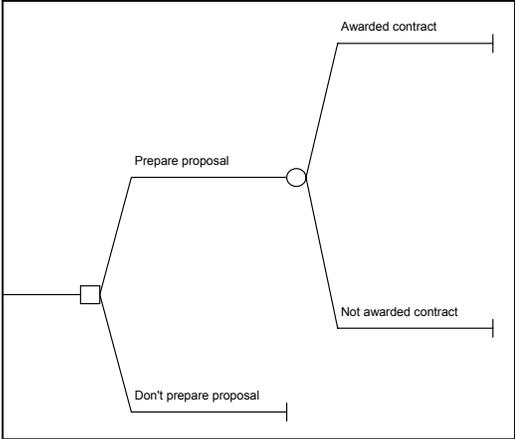
If DriveTek is awarded the contract, they must decide which approach to use. For the electronic and magnetic approaches, the result is uncertain, as shown in Figure 14.3. The arrangement of the decision and event branches is called the *structure* of the decision tree.

Figure 14.3 DriveTek Decisions and Events (Structure)
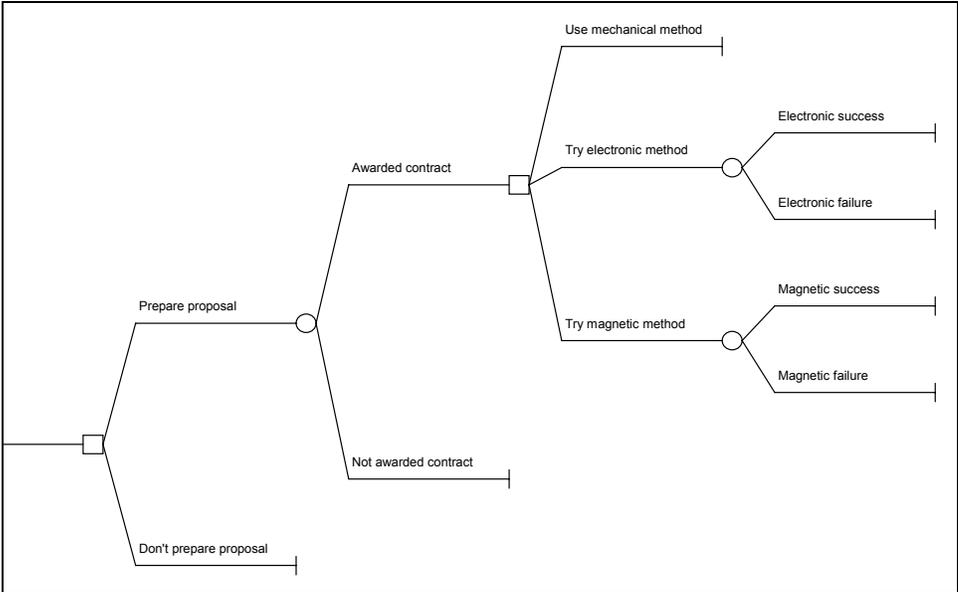
For representing a sequential decision problem, the tree diagram is usually better than the written description. In some decision problems, the choice may be obvious by looking at the diagram. That is, the decision maker may know enough about the desirability of the outcomes (endpoints in the tree) and how likely they are. But usually the next step in the analysis after documenting the structure is to assign values to the endpoints.

14.2 DECISION TREE TERMINAL VALUES

Each terminal node has an associated terminal value, sometimes called a payoff value, outcome value, or endpoint value. Each terminal value measures the result of a *scenario*: the sequence of decisions and events on a unique path leading from the initial decision node to a specific terminal node. To determine the terminal value, one approach assigns a cash flow value to each decision branch and event branch and then sums the cash flow values on the branches leading to a terminal node to determine the terminal value. Some problems require a more elaborate value model to determine the terminal values.

DriveTek Problem, Part B

DriveTek thinks it will cost $50,000 to prepare a proposal. If they are awarded the contract, DriveTek will receive an immediate payment of $250,000. The engineers think that the sure-success mechanical method will cost $120,000. The possibly-successful electronic approach will cost $50,000, and the more-likely-successful magnetic approach will cost $80,000. In the DriveTek problem, these distinct cash flows associated with many of the decision and event branches are shown in Figure 14.4.
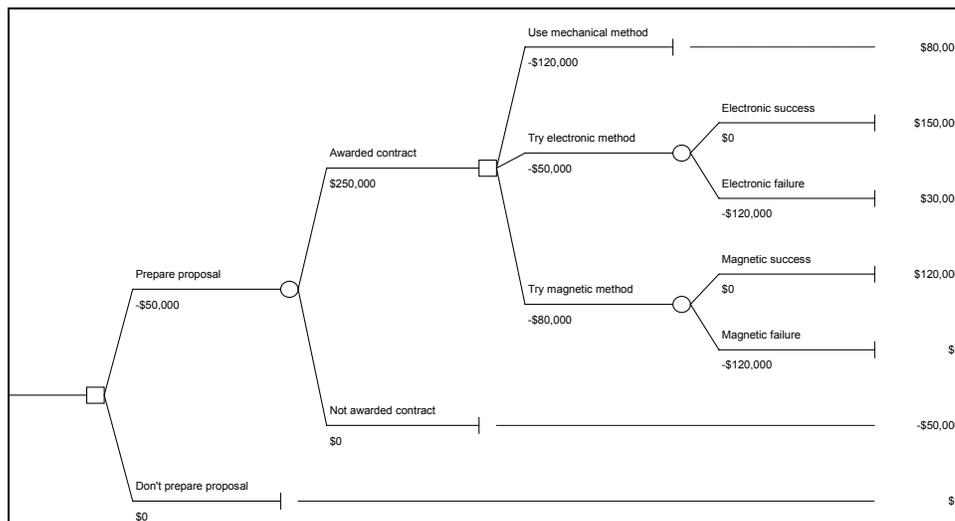
Figure 14.4 DriveTek Cash Flows and Outcome Values

Figure 14.4 also shows the sum of branch cash flows at the endpoints. For example, the $30,000 terminal value on the far right of the diagram is associated with the scenario shown in Figure 14.5.

Figure 14.5 Terminal Value for a Scenario

Branch Type	Branch Name	Cash Flow
Decision	Prepare proposal	−$50,000
Event	Awarded contract	+$250,000
Decision	Try electronic method	−$50,000
Event	Electronic failure (Use mechanical method)	−$120,000
	Terminal value =	+$30,000

14.3 DECISION TREE PROBABILITIES

DriveTek Problem, Part C

DriveTek management thinks there is a fifty-fifty chance of winning the contract. The engineers think that the inexpensive electronic method has only a 50% chance of satisfactory results. In their opinion the somewhat more costly magnetic method has 70% chance of success.

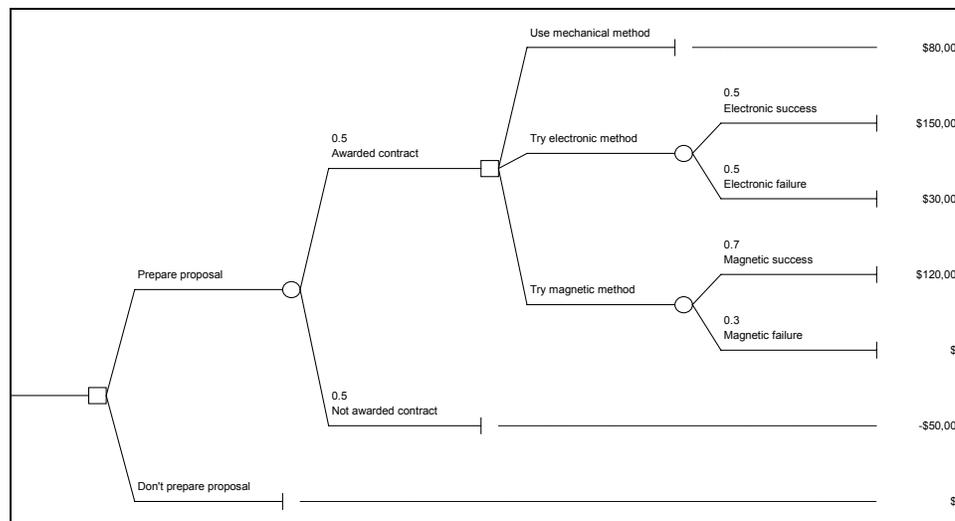**Figure 14.6** DriveTek Complete Decision Tree Model

Figure 14.6 is a complete decision tree model.

14.4 ROLLBACK METHOD

If we have a method for determining certain equivalents (expected values for a risk neutral decision maker), we don't need to examine every possible strategy explicitly. Instead, the method known as rollback determines the single best strategy.

The rollback algorithm, sometimes called backward induction or "average out and fold back," starts at the terminal nodes of the tree and works backward to the initial decision node, determining the certain equivalent rollback values for each node. Rollback values are determined as follows:

- At a terminal node, the rollback value equals the terminal value.

- At an event node, the rollback value for a risk neutral decision maker is determined using expected value (probability-weighted average); the branch probability is multiplied times the successor rollback value, and the products are summed.

- At a decision node, the rollback value is set equal to the highest rollback value on the immediate successor nodes.

In TreePlan tree diagrams the rollback values are located to the left and below each decision, event, and terminal node. Terminal values and rollback values for the DriveTek problem are shown below.

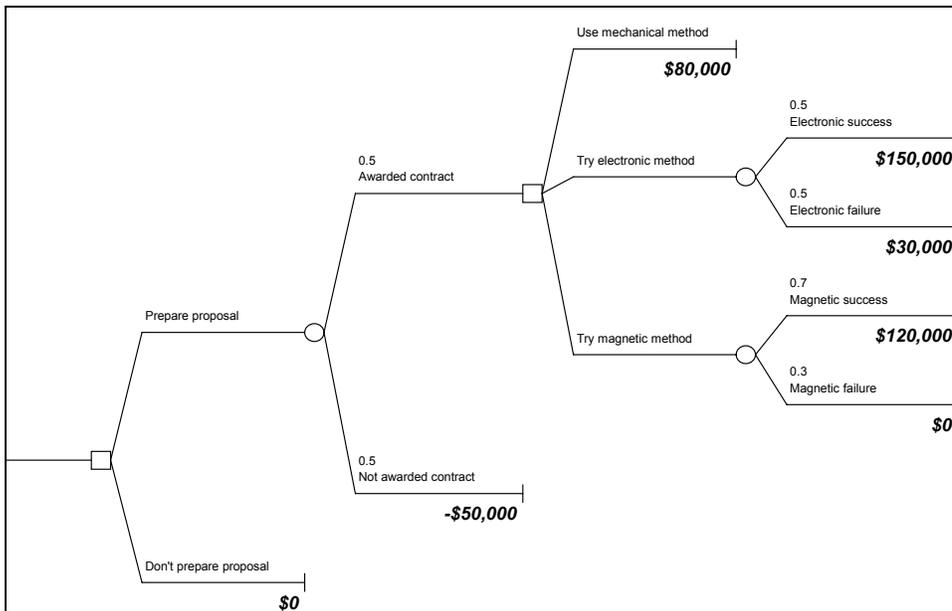**Figure 14.7** Terminal Node Rollback Values

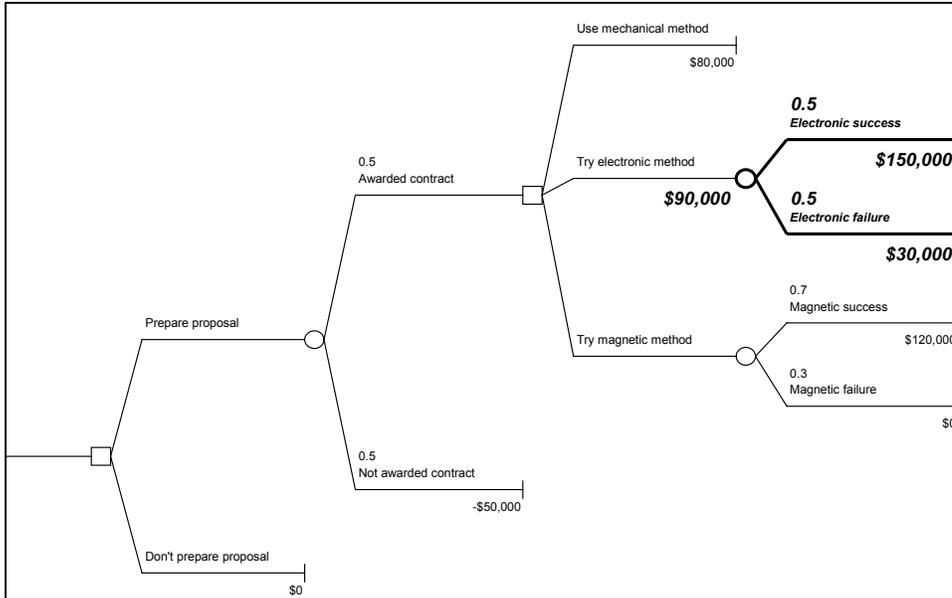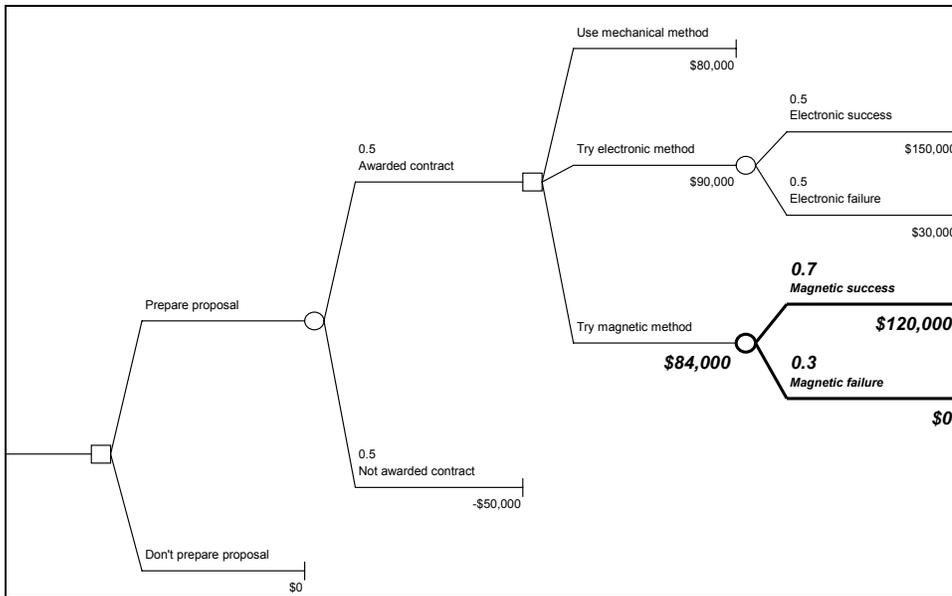**Figure 14.8** Electronic Event Node Rollback Value

Figure 14.9 Magnetic Event Node Rollback Value

At the event node following "Try electronic method," compute an expected value (probability-weighted average):

= 0.5*150,000 + 0.5*30,000
= 75,000 + 15,000
= 90,000

At the event node following "Try magnetic method," compute an expected value:
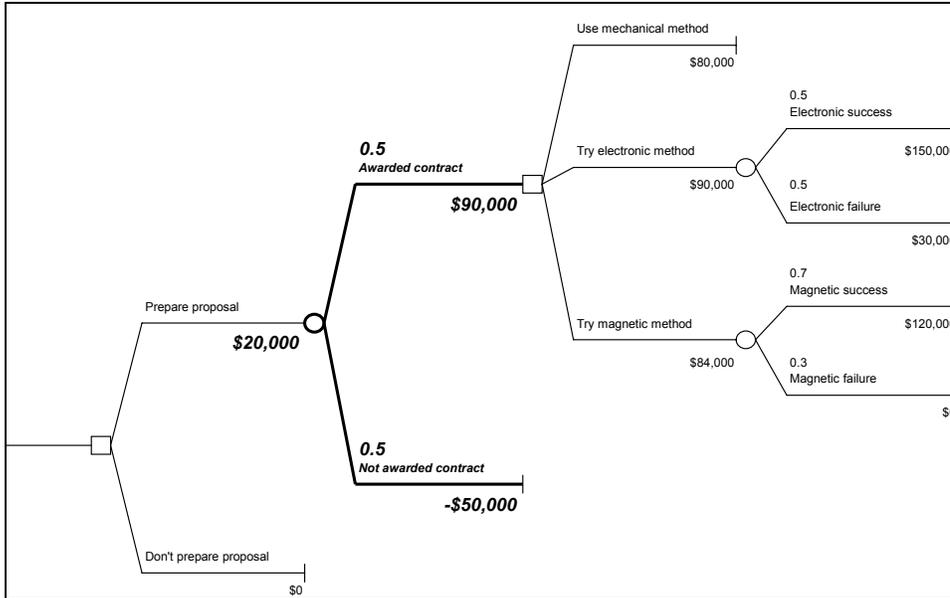
= 0.7*120,000 + 0.3*0
= 84,000 + 0
= 84,000

Figure 14.10 Method Decision Node Rollback Value
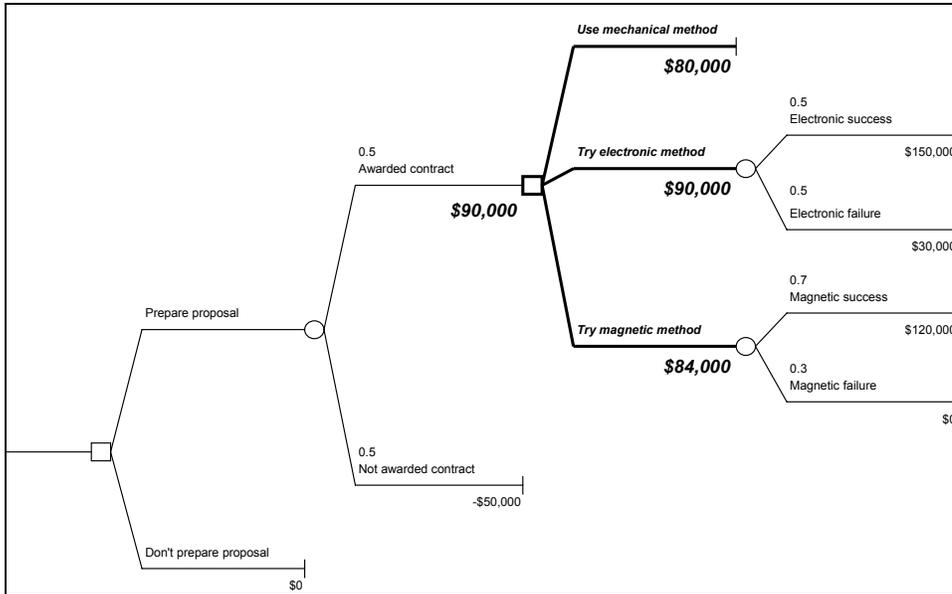
At the decision node following "Awarded contract," choose the highest successor rollback value:

= Max($80,000, $90,000, $84,000)
= $90,000

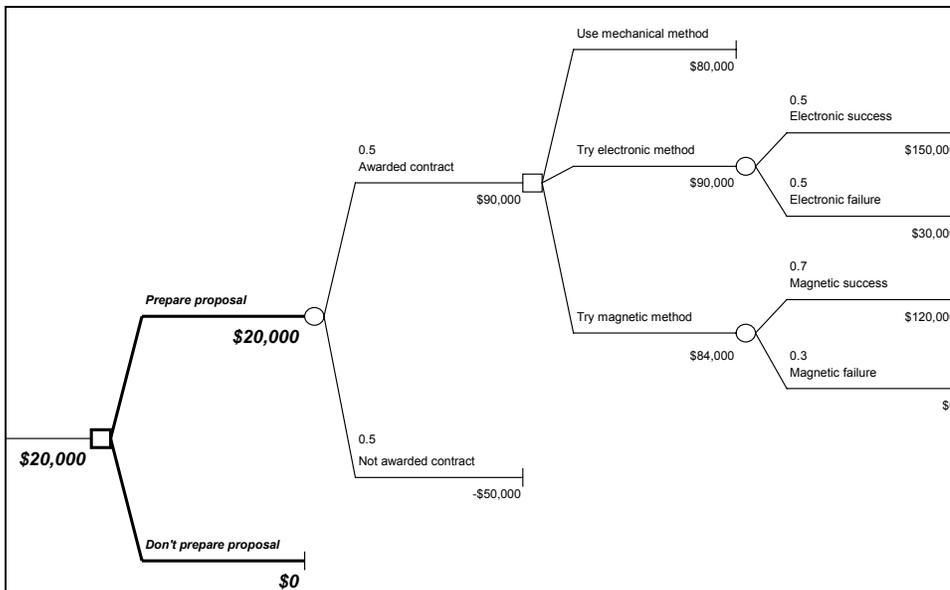**Figure 14.11** Contract Event Node Rollback Value

At the event node following "Prepare proposal," compute an expected value:

$$= 0.5*90,000 + 0.5*(-50,000)$$
$$= 45,000 + (-25,000)$$
$$= 20,000$$

Figure 14.12 Proposal Decision Node Rollback Value
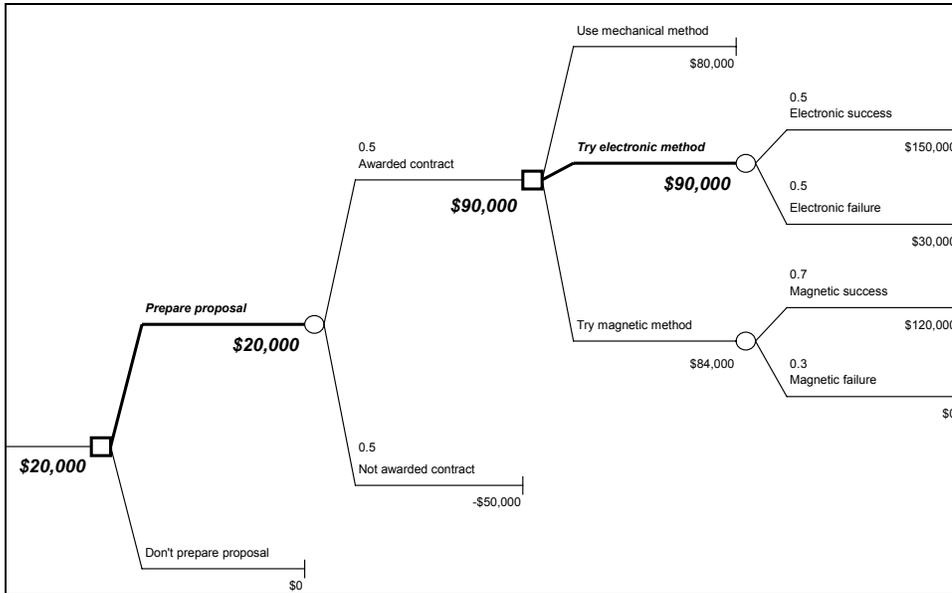
At the initial decision node, choose the highest successor rollback value:

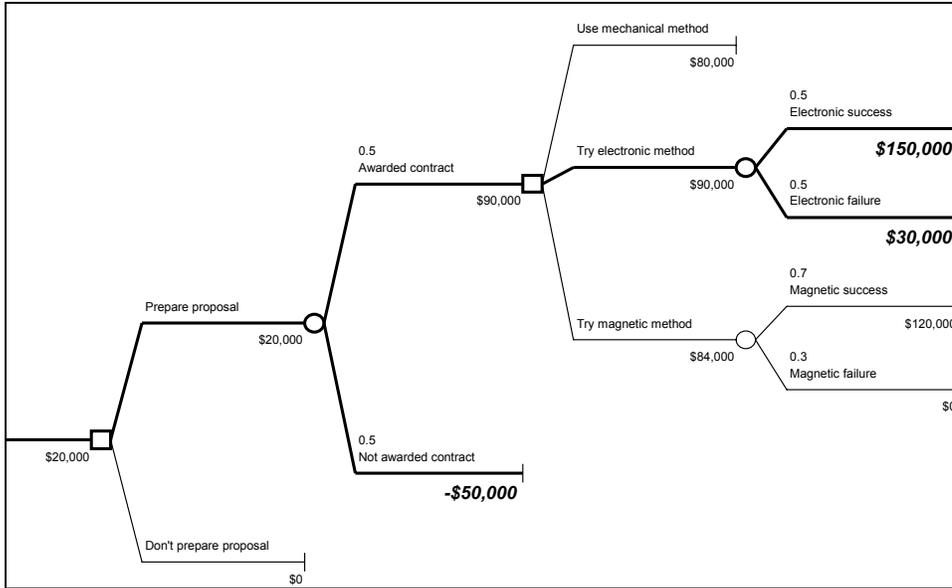= Max($20,000, $0)
= $20,000

Figure 14.13 Rollback Optimal Choices

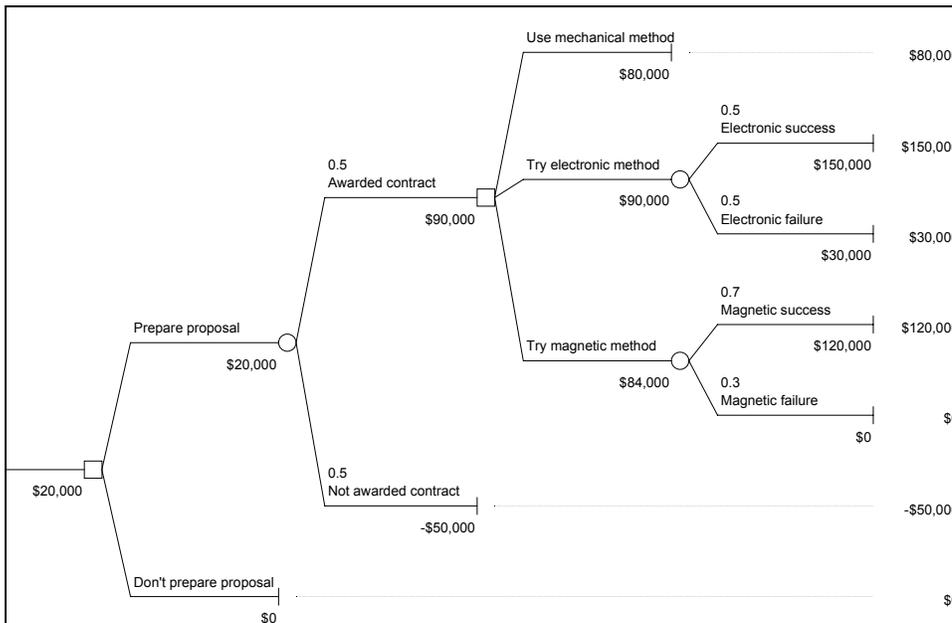At the initial decision node, choose "Prepare proposal."

If "Awarded contract," choose "Try electronic method."

Figure 14.14 Rollback Optimal Strategy Branches and Payoffs

Using the best choices at decision nodes (controllable) and depending on what happens at the event nodes (uncontrollable), there are three possible outcomes.

Figure 14.15 All Rollback Values

Optimal Strategy

After the rollback method has determined certain equivalents for each node, the optimal strategy can be identified by working forward through the tree. At the initial decision node, the $20,000 rollback value equals the rollback value of the "Prepare proposal" branch, indicating the alternative that should be chosen. DriveTek will either be awarded the contract or not; there is a subsequent decision only if DriveTek obtains the contract. (In a more complicated decision tree, the optimal strategy must include decision choices for all decision nodes that might be encountered.) At the decision node following "Awarded contract," the $90,000 rollback value equals the rollback value of the "Try electronic method" branch, indicating the alternative that should be chosen. Subsequently, if the electronic method fails, DriveTek must use the mechanical method to satisfy the contract.

14.5 NEWOX DECISION TREE PROBLEM

The Newox Company is considering whether or not to drill for natural gas on its own land. If they drill, their initial expenditure will be $40,000 for drilling costs. If they strike gas, they must spend an additional $30,000 to cap the well and provide the necessary hardware and control equipment. (This $30,000 cost is not a decision; it is associated with the event "strike gas.") If they decide to drill but no gas is found, there are no other subsequent alternatives, so their outcome value is $-40,000.

If they drill and find gas, there are two alternatives. Newox could sell to West Gas, which has made a standing offer of $200,000 to purchase all rights to the gas well's production (assuming that Newox has actually found gas). Alternatively, if gas is found, Newox can decide to keep the well instead of selling to West Gas; in this case Newox manages the gas production and takes its chances by selling the gas on the open market.

At the current price of natural gas, if gas is found it would have a value of $150,000 on the open market. However, there is a possibility that the price of gas will rise to double its current value, in which case a successful well will be worth $300,000.

The company's engineers feel that the chance of finding gas is 30 percent; their staff economist thinks there is a 60 percent chance that the price of gas will double.