Learning Analytics

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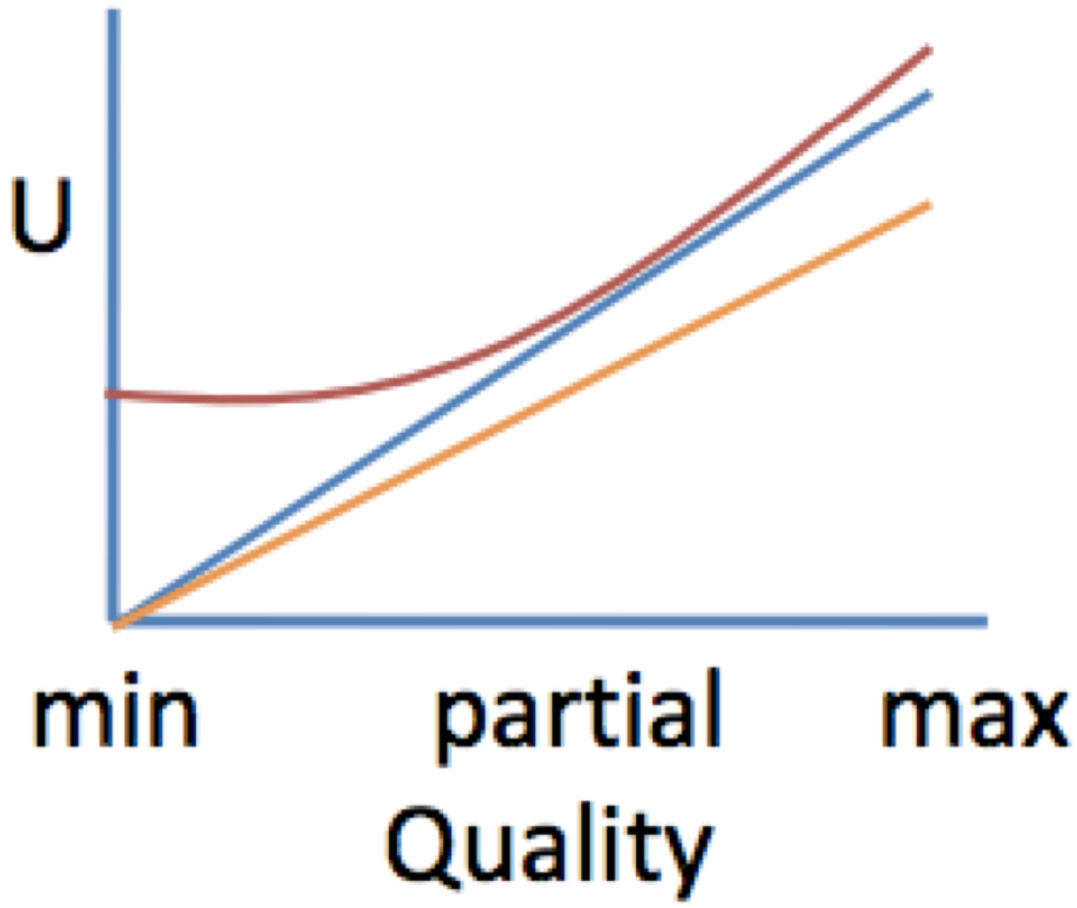
Today: Eliciting a Utility Function



Previous Lectures

- Probabilistic inference (Bayes nets)
  - Maintain beliefs about the world
  - Probabilities can be estimated

- Utility theory
  - Quantify preferences
  - Where do utilities come from?

- Combination:
  - Take actions with maximum expected utility

Preference Over Lotteries

- Recall: action/choice outcomes can be stochastic
- Each action is a “lottery” over outcomes

- A simple lottery over X has the form:

$$l = [(p_1,x_1), (p_2,x_2), \ldots, (p_n,x_n)]$$

where $p_i \geq 0$ and $\sum p_i = 1.0$

- Outcomes are just trivial lotteries
  - One outcome has probability 1.0

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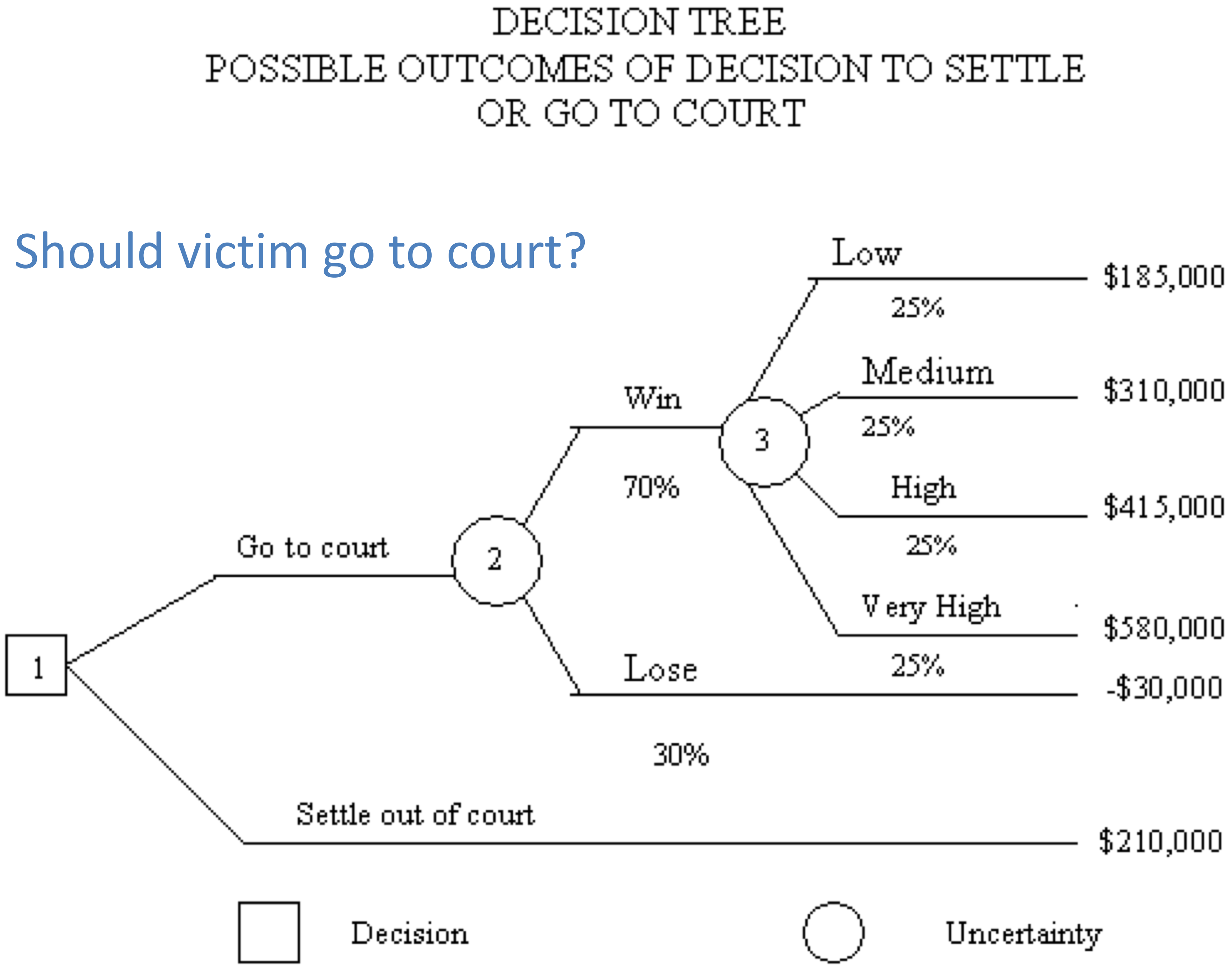
  where $p_i \geq 0$ and $\sum p_i = 1.0$

- A compound lottery allows outcomes to be lotteries:

  $$[(p_1,l_1), (p_2,l_2), \ldots, (p_n,l_n)]$$

  restrict to finite compounding

Should victim go to court?



DECISION TREE
POSSIBLE OUTCOMES OF DECISION TO SETTLE
OR GO TO COURT

Image taken from www.mediate.com

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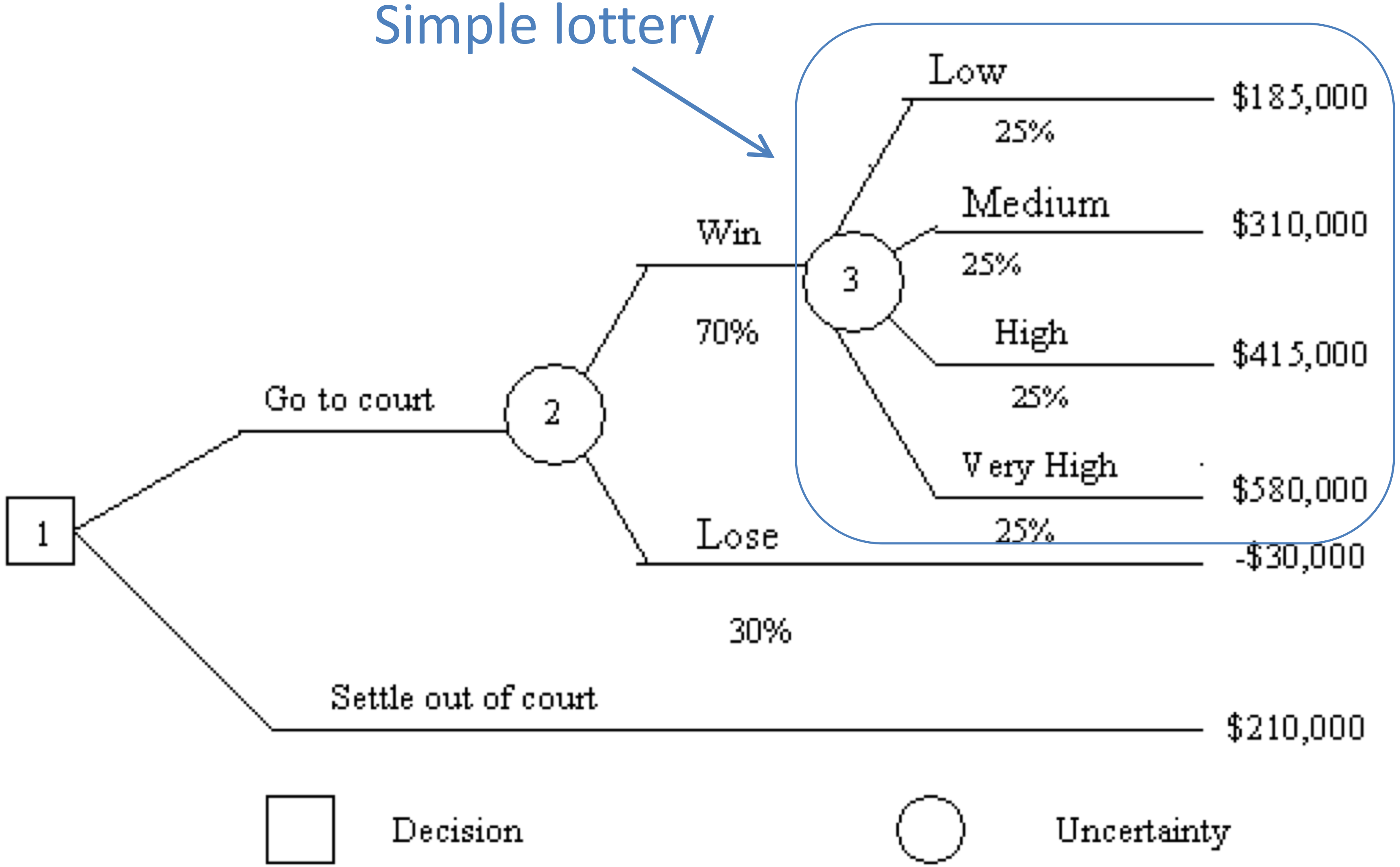


Simple lottery

Image taken from www.mediate.com

DECISION TREE
POSSIBLE OUTCOMES OF DECISION TO SETTLE OR GO TO COURT

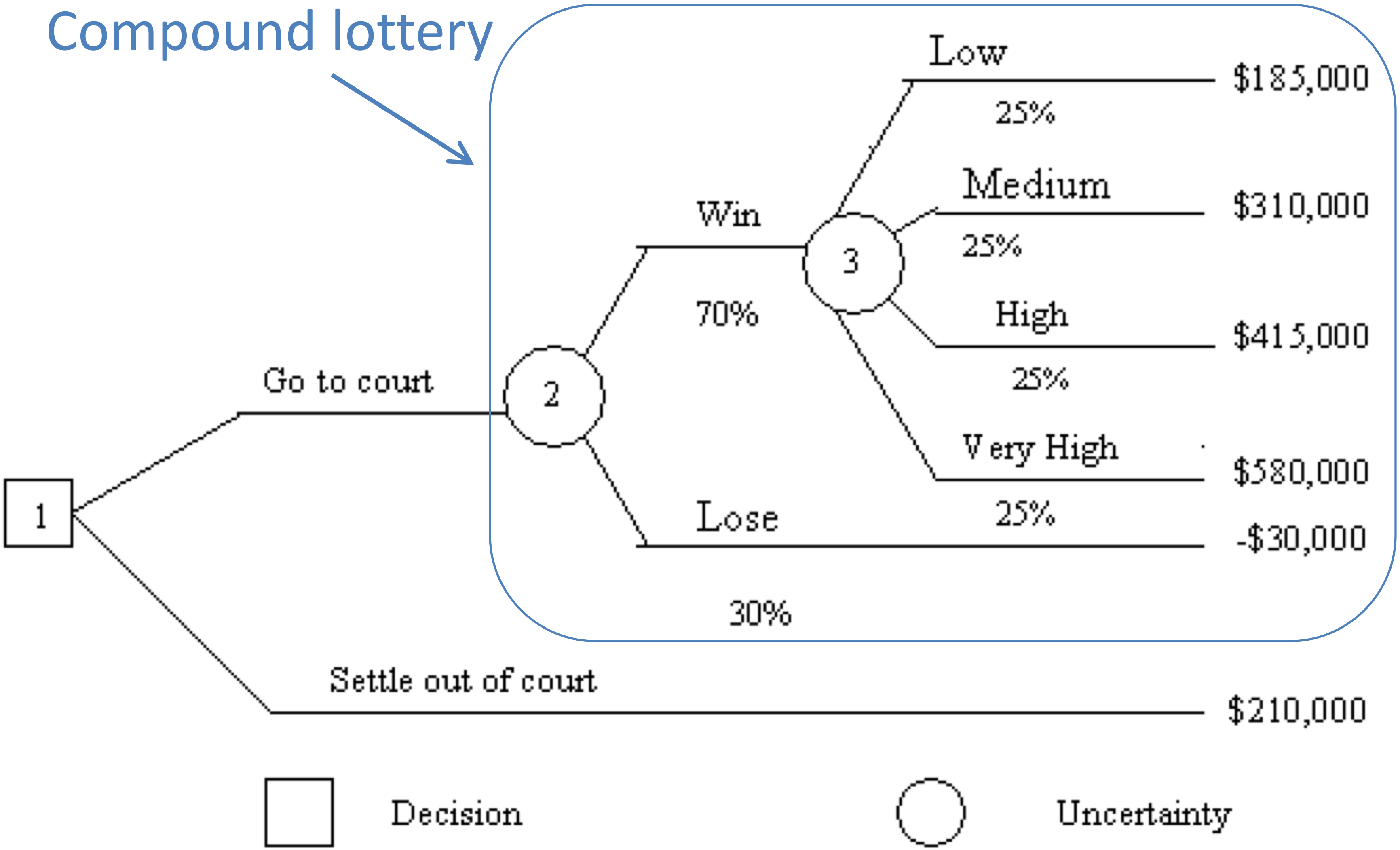

Compound lottery

Image taken from www.mediate.com

Which Would You Choose?

- Between:
  - [(0.5, $200,000), (0.5 $0)]　　　Gamble

Which Would You Choose?

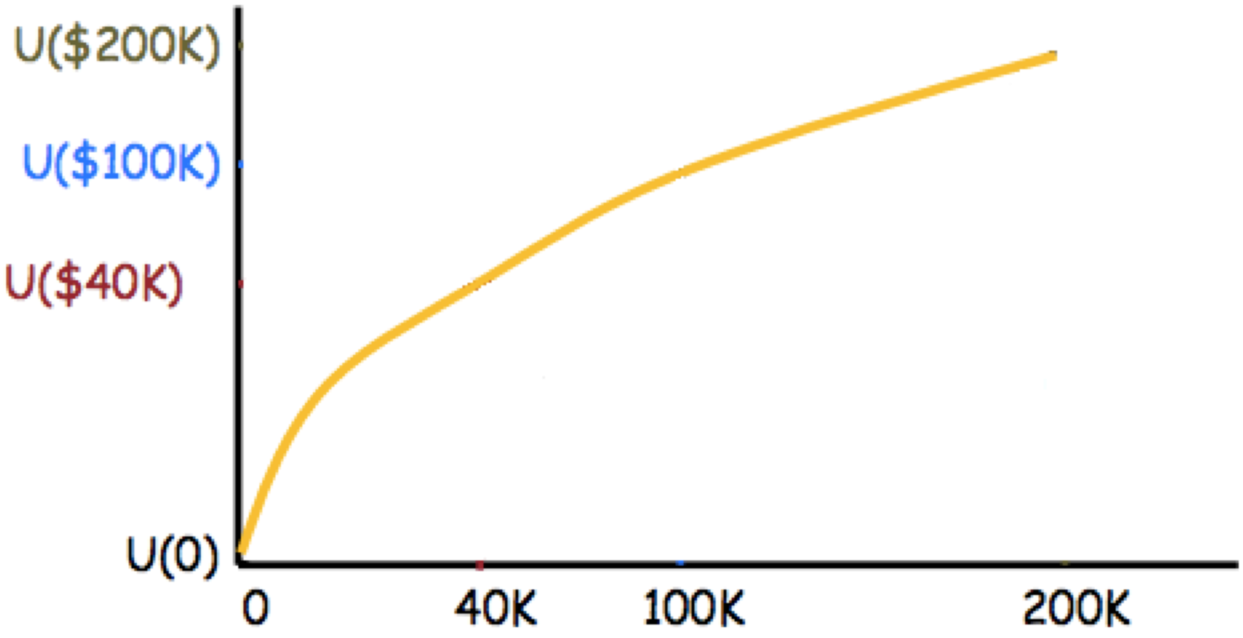
- Between:
  - [(0.5, \$200,000), (0.5 \$0)] Gamble
  - \$100,000 Sure Win

Which Would You Choose?

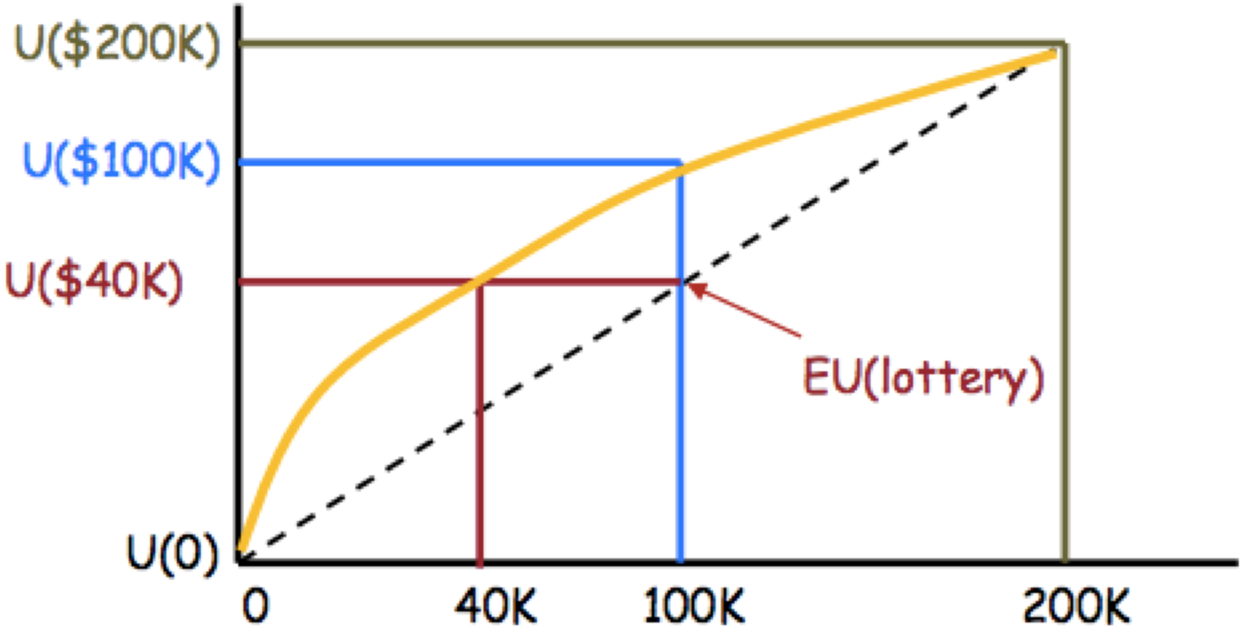
- Between:
  - [(0.5, \$200,000), (0.5 \$0)]
  - \$100,000

- Utility of money is nonlinear!
  - U(\$100K) > 0.5*U(\$200K) + 0.5*U(\$0)
  - What if lottery changed to: \$500K, \$1M,… p = 0.6, 0.7, 0.999, …?

Concave Utility Function



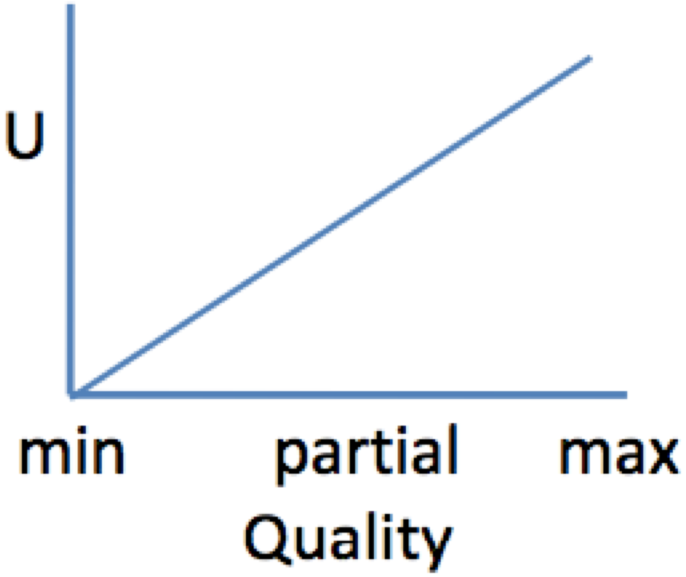
Concave Utility Function



Most people take a sure win of $40K over a gamble for $200K

Abstract Utility Functions for Intelligent Tutoring Systems

- How valuable is the help you get as a function of …?



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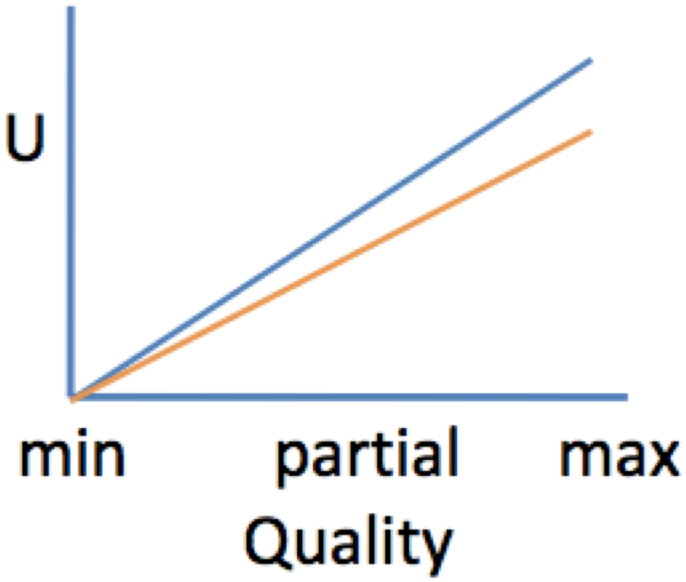

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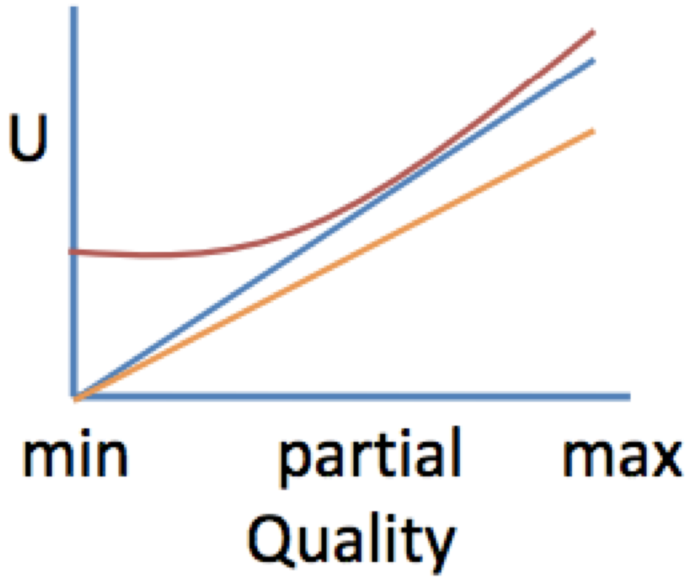

Each function may represent different individual users

Abstract Utility Functions for Intelligent Tutoring Systems

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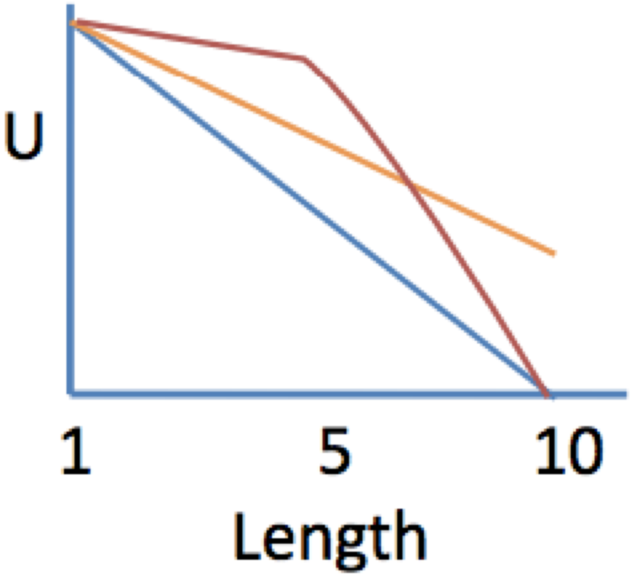

Recall from HCI: interface bloat is bad design

Abstract Utility Functions for Intelligent Tutoring Systems

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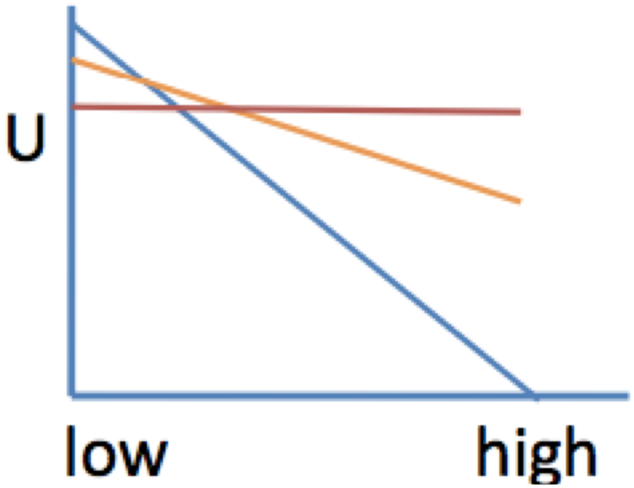

User Independence

Abstract Utility Functions for Intelligent Tutoring Systems

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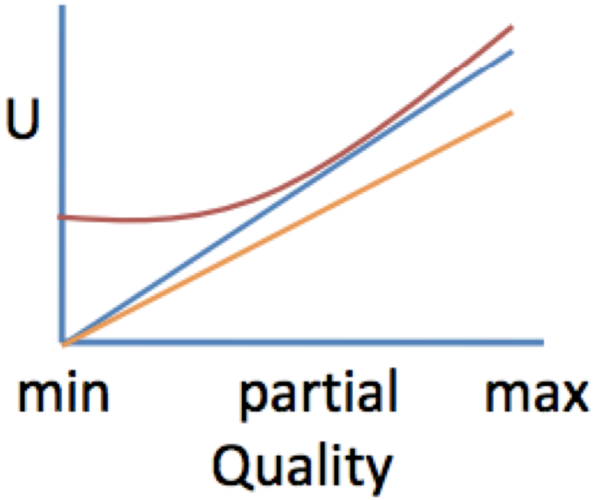

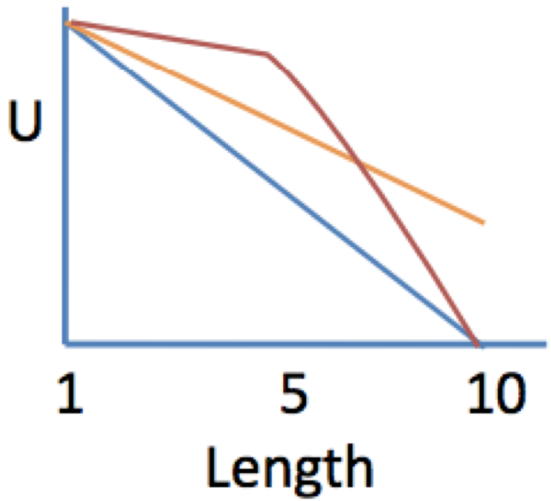

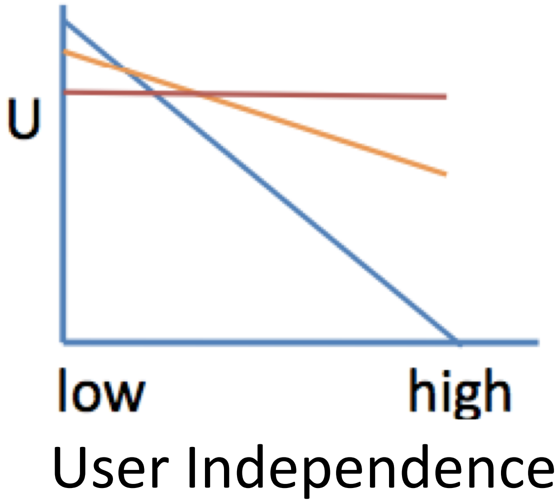

- Additive decomposition?
  - – Roughly: U(help) = c_1*quality + c_2*length + c_3*indep

Elicitation Queries

- Decision problem setup:
  - Best outcome with utility 1.0
  - Worst outcome with utility 0.0
  - Other outcomes have utility values in [0,1]

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- Standard gamble
  - SG(pr) = [pr,o^{best}; 1-pr, o_{worst}]
  - Expected utility of SG = pr

Which would you choose:

- [(0.5, $200,000), (0.5 $0)]
- $100,000

Elicitation Queries

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Which would you choose:

- [(0.5, \$200,000), (0.5 \$0)]
- \$100,000

- Example:
  - Let pr = 0.5, then EU(SG) = 0.5(1) + 0.5(0) = 0.5
  - Let pr = 0.8, then EU(SG) = 0.8(1) + 0.2(0) = 0.8

What This Means

- Given an outcome
  - Call this o_i
- You want to know its utility value in [0,1]
  - Call this pr
- Can’t directly ask for pr
- Set it up a standard gamble with two options:
  - $SG(pr) = [pr, o^{best}; 1\text{-}pr, o_{worst}]$
  - Sure win of o_i
- Find the value of pr such that the person is indifferent between the two options
  - Then pr is the expected value of o_i

Standard Gamble Query

- Standard gamble
  - – SG(pr) = [pr,o^{best}; 1-pr, o_{worst}]
  - – Expected utility of SG = pr

- Standard gamble query
  - – Alternative A: SG(pr)
  - – Alternative B: o_i
  - – Question to ask: What is *pr* s.t. *SG(pr) = o_i*?
  - – Response to elicit from user: pr

Try It Yourself

- Suppose Quiz 2:
  - Choose between coin flip and a fixed score

- Find a partner and ask:
  - Alternative A: SG(pr) = [pr, 100%; 1-pr, 0%]
  - Alternative B: o_i = 51%
  - Question to ask: What is *pr* s.t. $SG(pr) = o_i$?
  - Response to elicit from user: pr

- Switch roles and replace:
  - Alternative B: o_i = 88%

How well did it go?

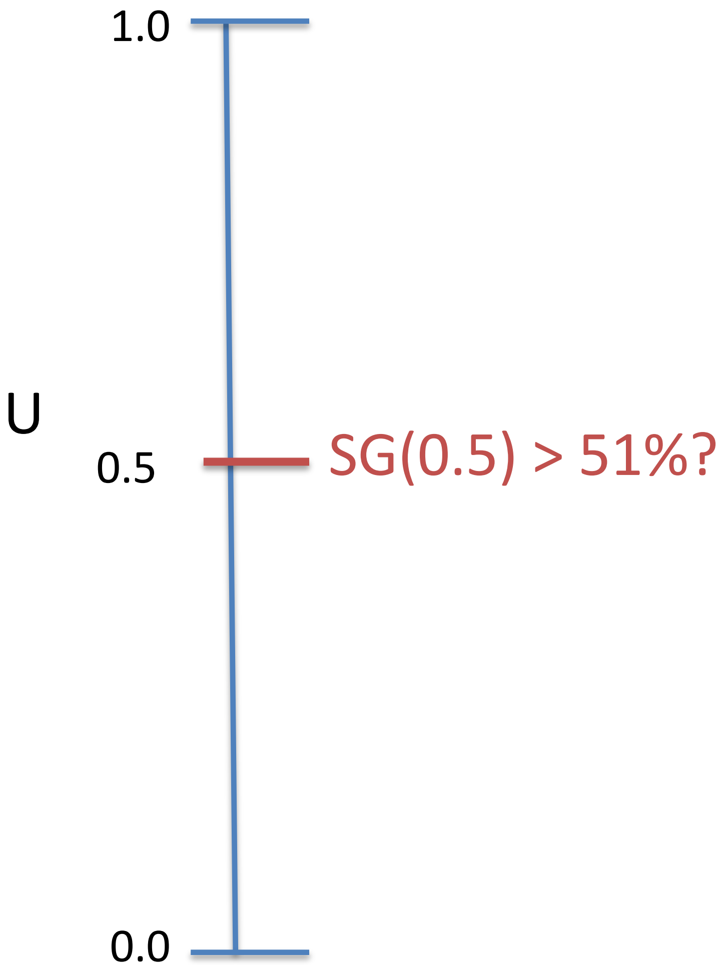
Bound Query

- Bound query
  - Alternative A: SG(pr)
  - Alternative B: o_i
  - Question to ask: Given *pr*, is *SG(pr)* > o_i?
  - Response to elicit from user: Yes/No

Like binary search

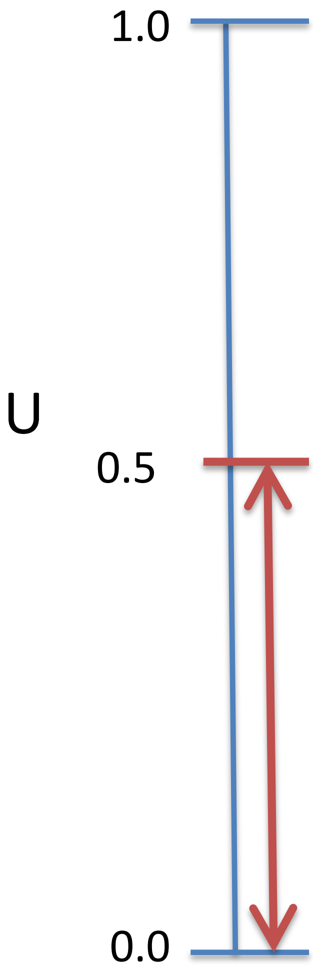
- Less information gain
  - Must ask sequence of queries to identify *pr* of o_i

[pr,100%;1-pr,0%] > 51% ?



SG(0.5) > 51%?

[pr,100%;1-pr,0%] > 51% ?



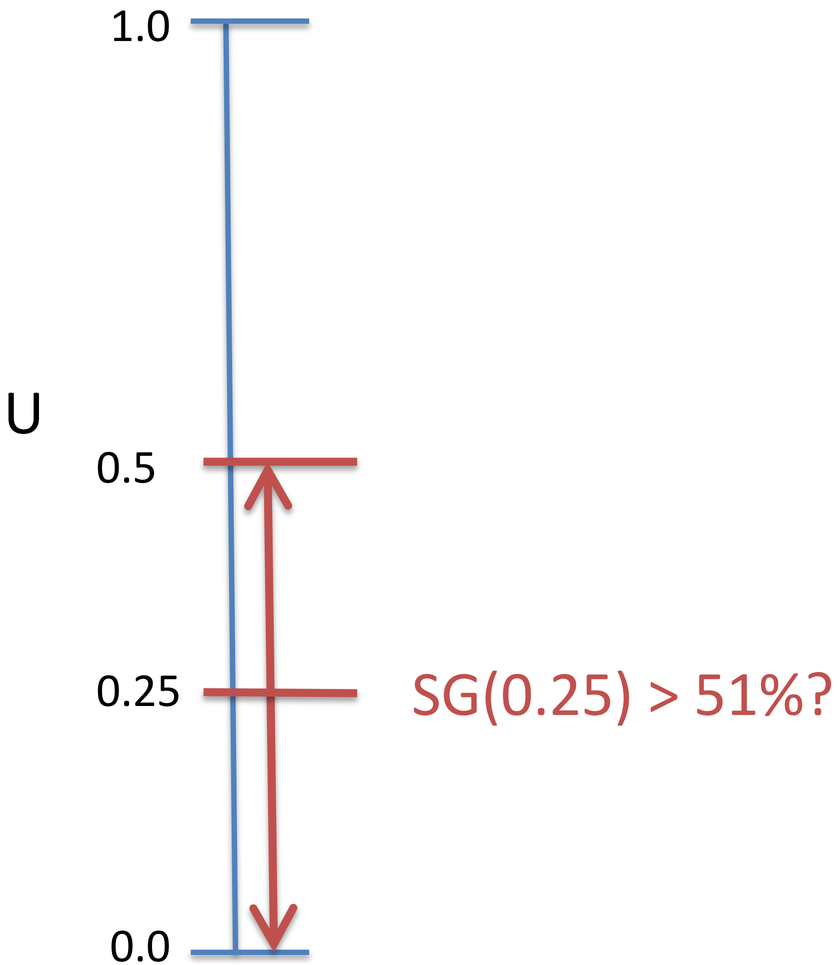
SG(0.75) > 51%?

Must still be “yes”

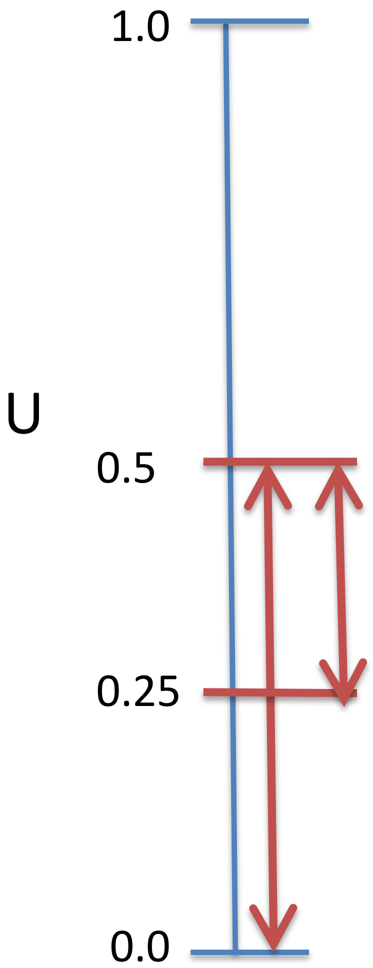
SG(0.5) > 51%? **yes**

- Search lower region

[pr,100%;1-pr,0%] > 51% ?



[pr,100%;1-pr,0%] > 51% ?

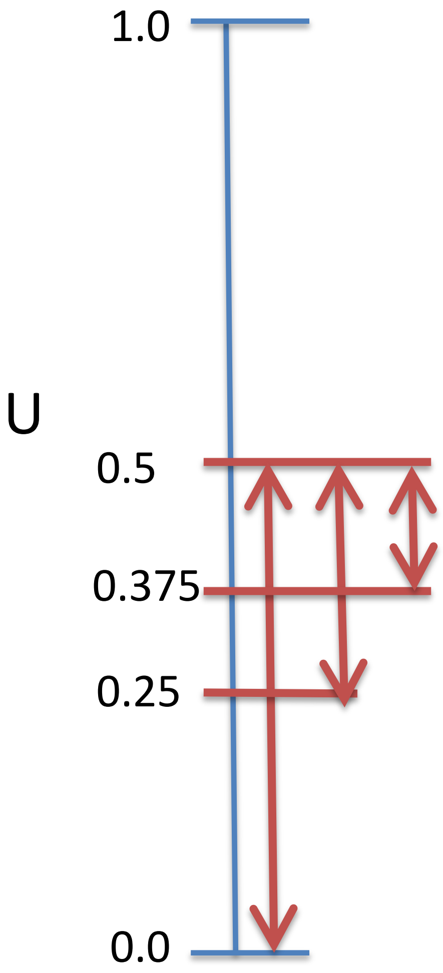


SG(0.25) > 51%? **no**

- Search upper region

SG(0.10) > 51%?
Must still be “no”

[pr,100%;1-pr,0%] > 51% ?



SG(0.375) > 51%? **no**

- Search upper region

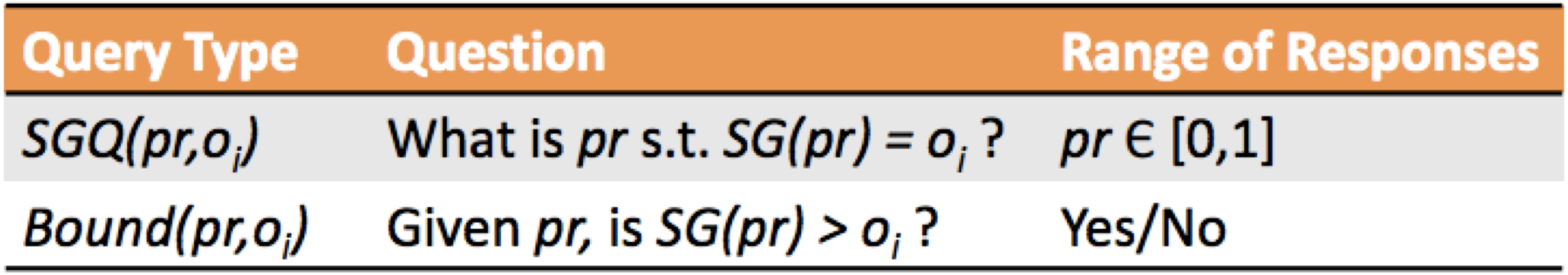
Continue until the response is "the two options are the same"

Try The New Version

- Same context (Quiz 2 score)
  - Construct query (choose reasonable *pr*)
  - Keep asking queries until you've identified *pr*

- Find a partner and ask:
  - Alternative A: SG(pr) = [pr, 100%; 1-pr, 0%]
  - Alternative B: o_i = 75%
  - Question to ask: Given *pr*, is $SG(pr) > o_i$?
  - Response to elicit from user: Yes/No

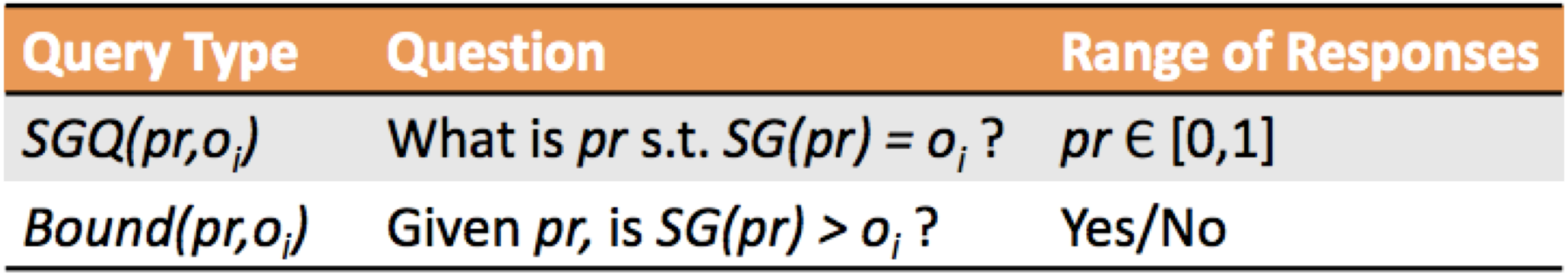
Comparison



Query Type	Question	Range of Responses
SGQ(pr,o_i)	What is *pr* s.t. *SG(pr)* = o_i ?	*pr* ∈ [0,1]
Bound(pr,o_i)	Given *pr*, is *SG(pr)* > o_i ?	Yes/No

- Which is better?

Comparison



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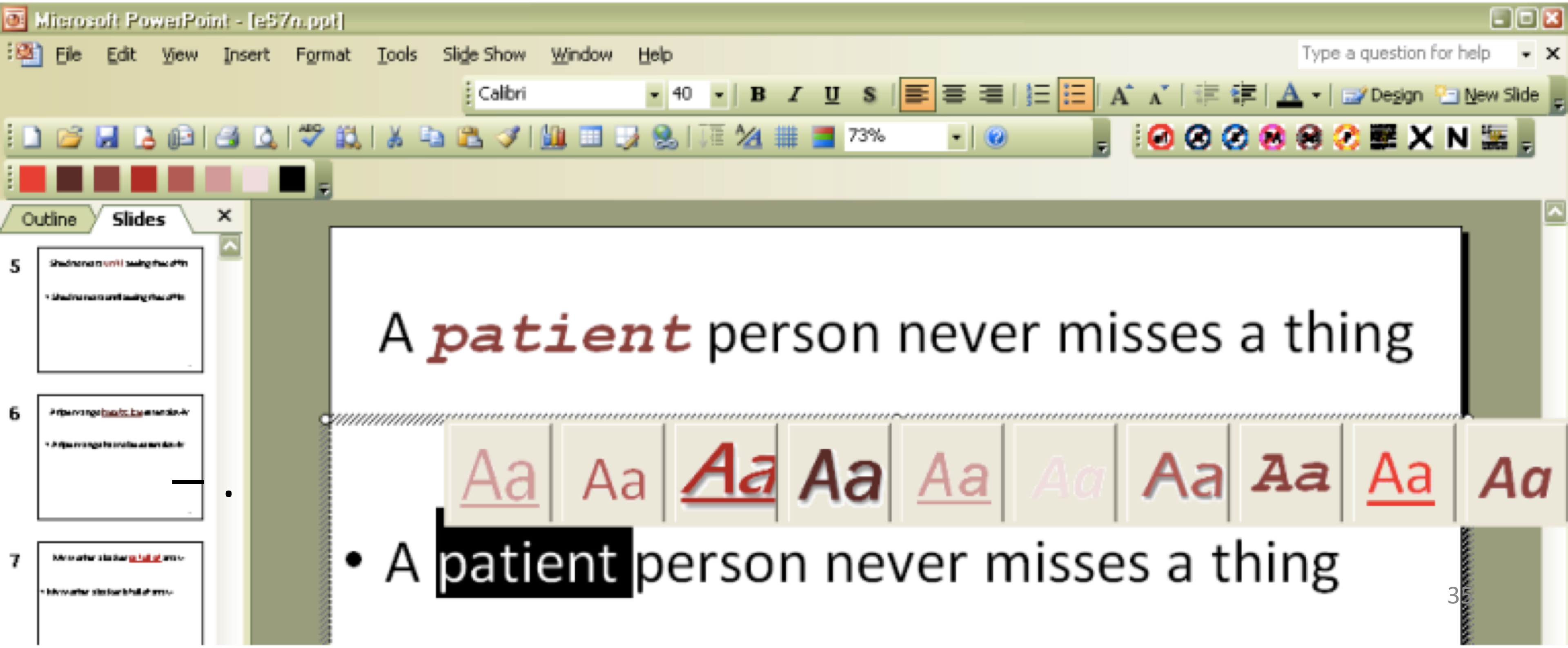
SGQ(pr,o_i)	What is *pr* s.t. *SG(pr)* = o_i ?	*pr* ∈ [0,1]
Bound(pr,o_i)	Given *pr*, is *SG(pr)* > o_i ?	Yes/No

- Which is better?
  - – SGQ provides pinpoint answer, but mixture of outcomes is difficult to interpret
  - – Bound query offers more meaningful queries, but less meaningful as feasible regions get smaller

Case Study in Software Help (Hui & Boutilier 2008)

- Help: Macro suggestion in PowerPoint toolbar
- Variables: Length, Quality, User Independence
  - User independence proxy via task difficulty



Microsoft PowerPoint - [e57n.ppt]
File Edit View Insert Format Tools Slide Show Window Help
Type a question for help
Calibri 40
Design New Slide
73%
Outline Slides
5
6
7
Aa

A *patient* person never misses a thing

– .

- A patient person never misses a thing

Experiment Setup

- Series of controlled highlighting task in PowerPoint

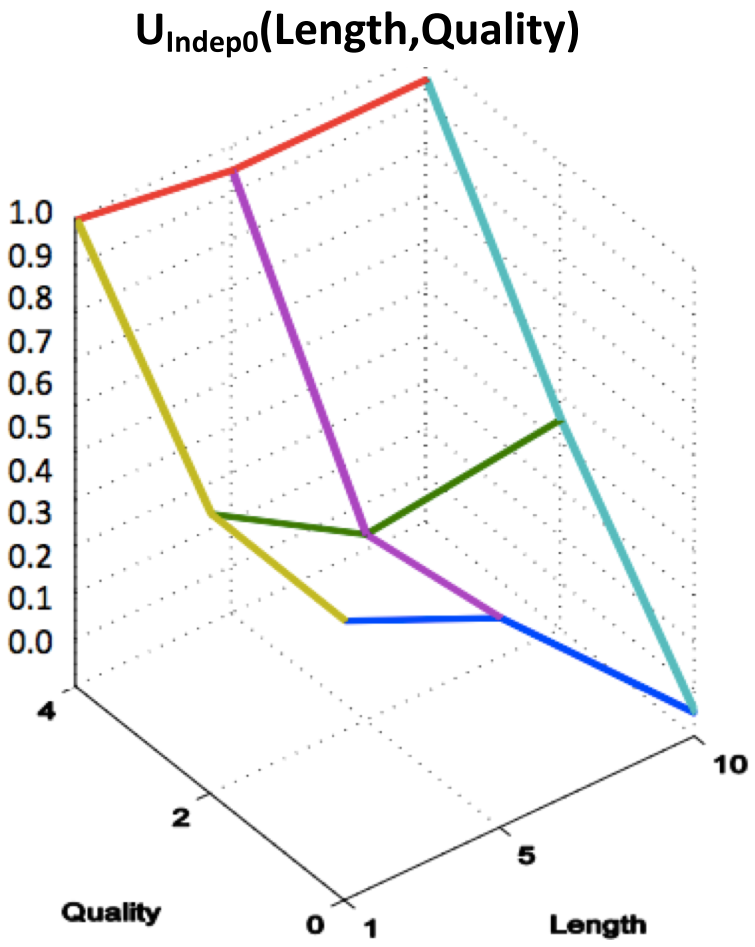
- Goal: To sample *U(I,L,Q)*
  - Indep, I = 0 (low), 1 (high)
  - Length, L = 1, 5, 10 icons
  - Quality, Q = 0 (wrong), 2 (partial), 4 (perfect)

- o^{best} = I0,L1,Q4
- o_{worst} = I1,L10,Q0

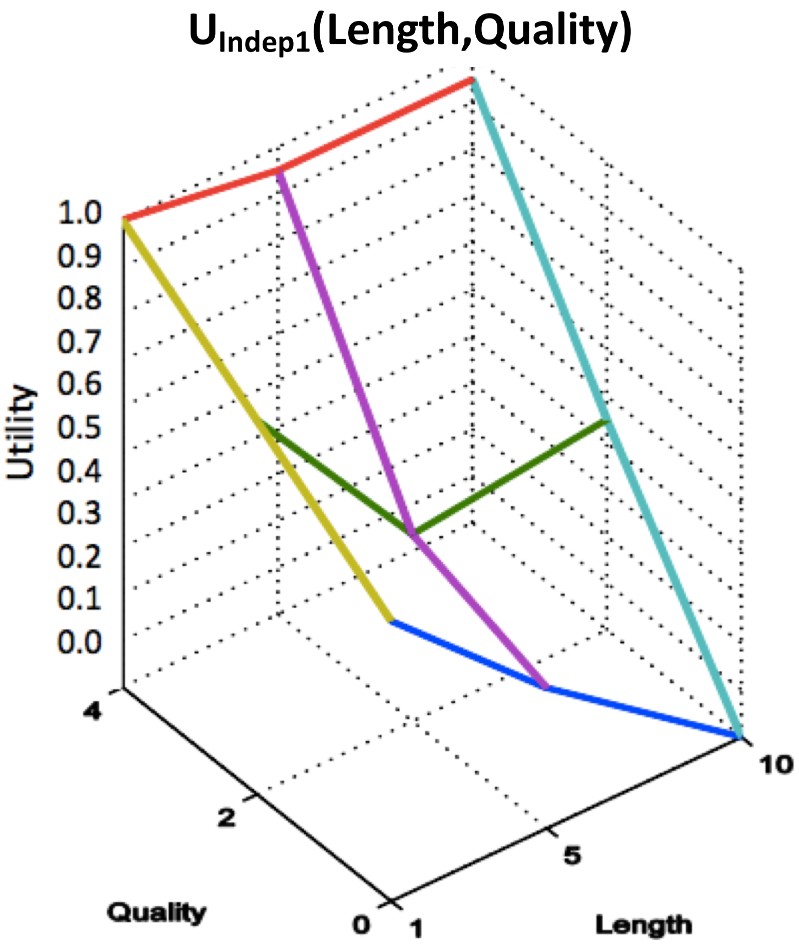
- Elicited until small “feasible” regions (pr ± 0.05)

User 11 (midpoints)

When: Task is difficult



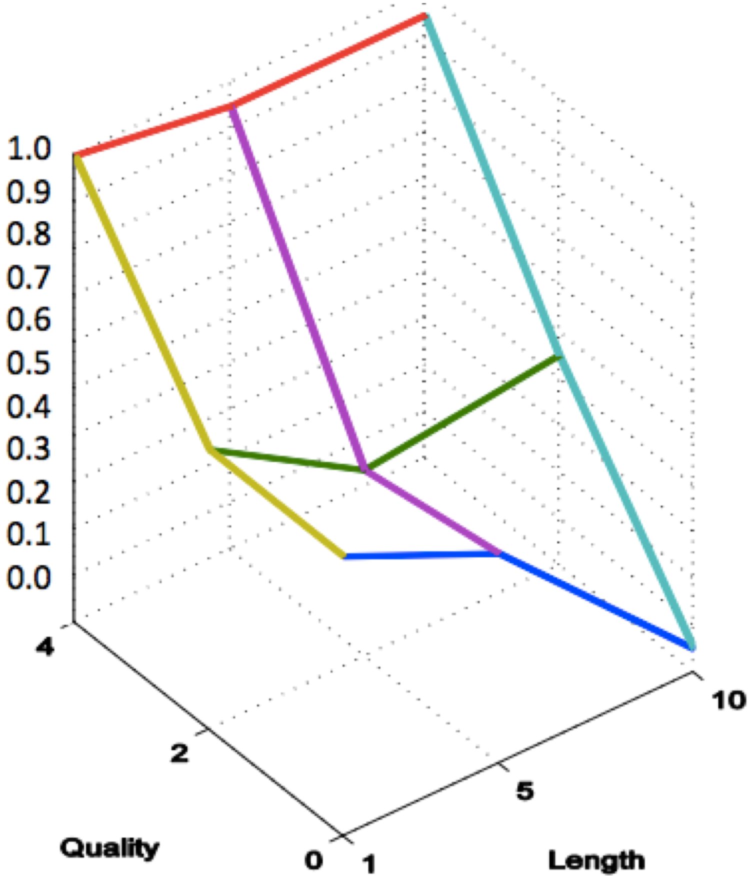
When: Task is easy



User 11 – role of length

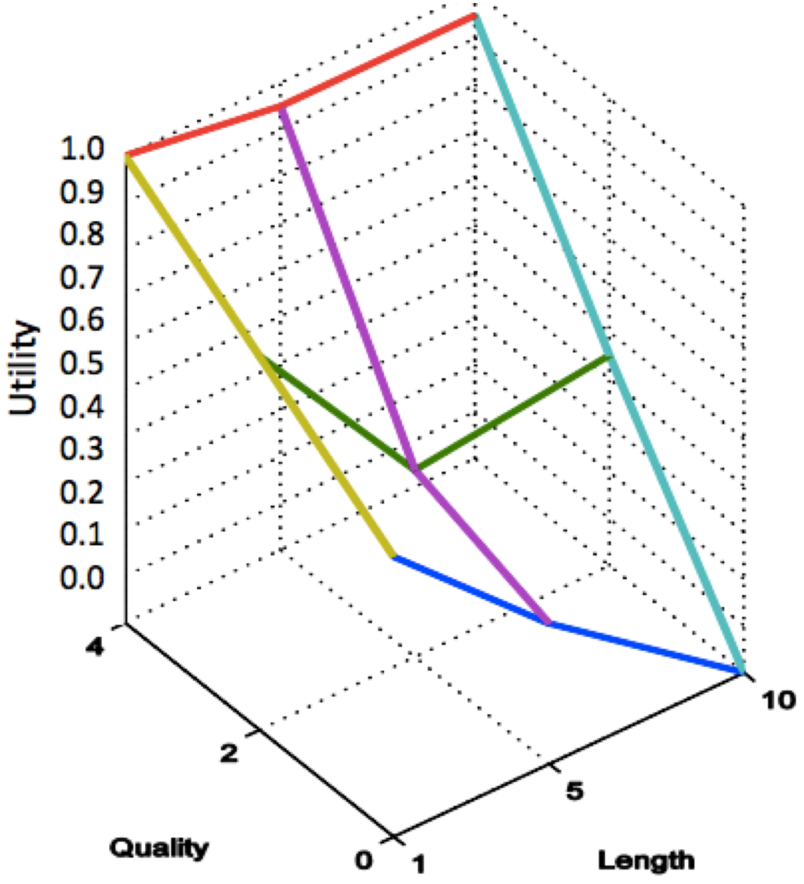
When: Task is difficult

U_{Indep0}(Length,Quality)



When: Task is easy

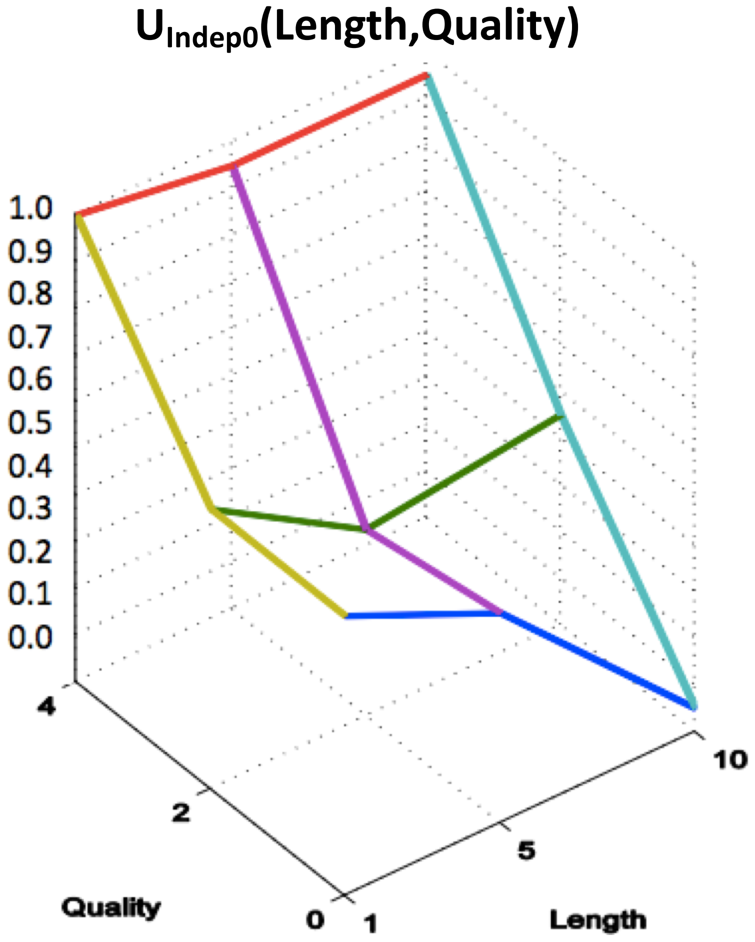
U_{Indep1}(Length,Quality)



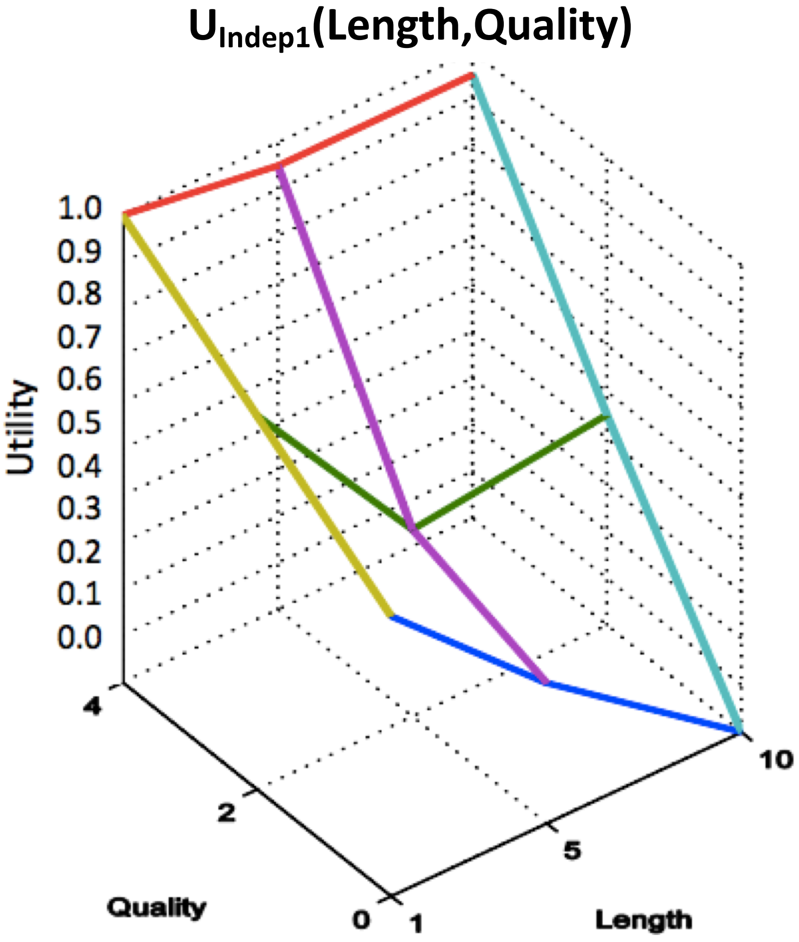
- High tolerance for length when Qual is also high

User 11 – value of non-perfect help

When: Task is difficult



When: Task is easy

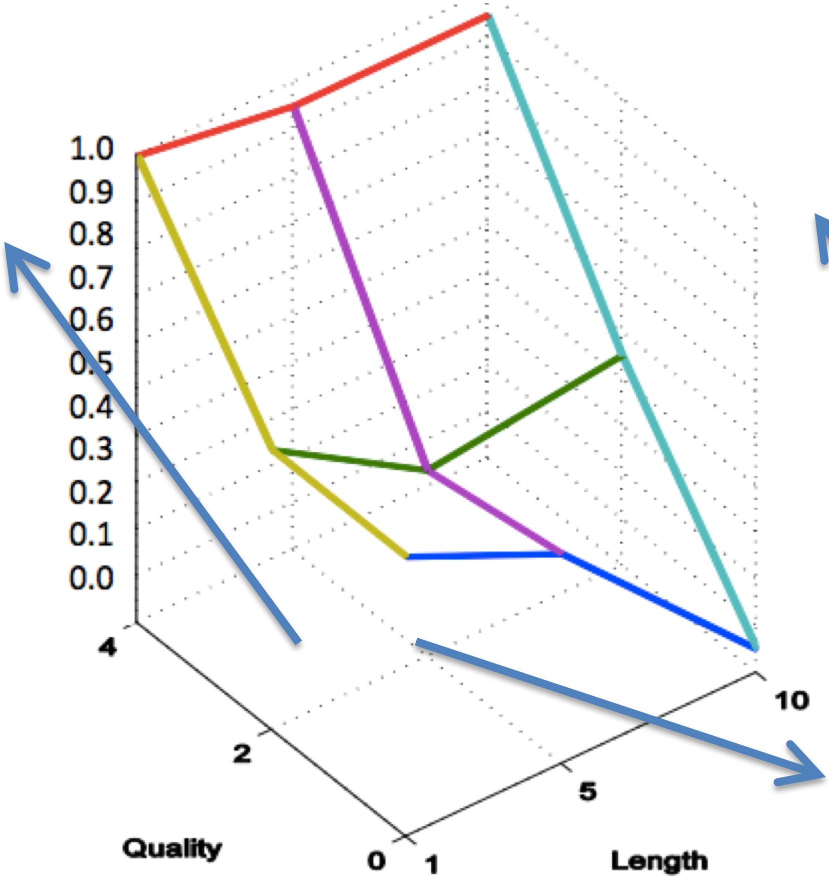


- Utility in Q2, even Q0
- Higher utility with easy task

User 11 – monotonicity

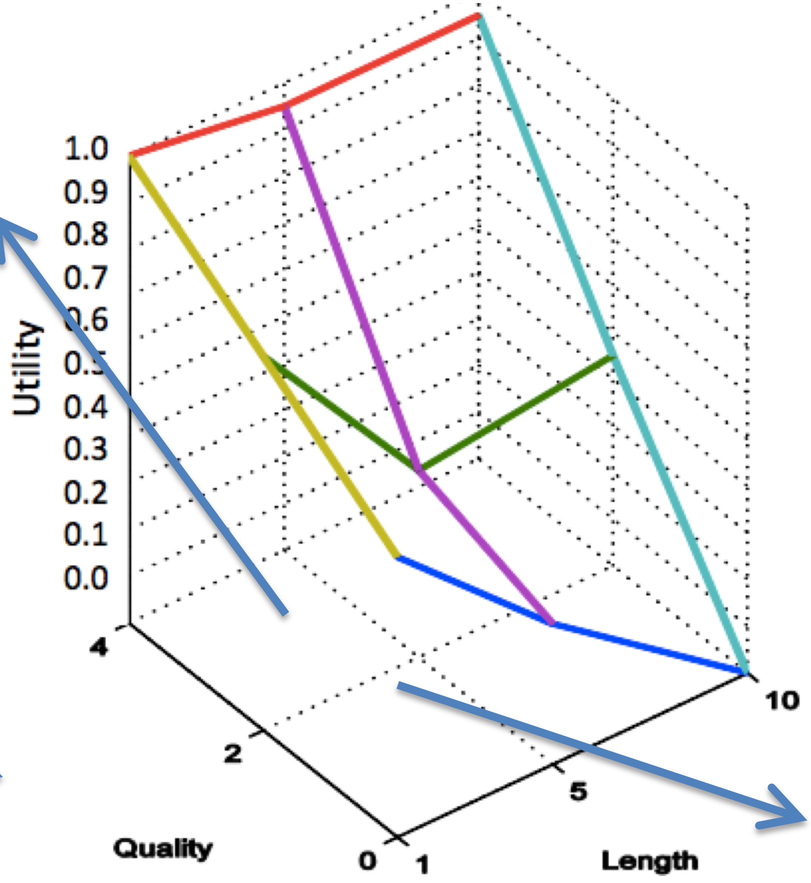
When: Task is difficult

U_{Indep0}(Length,Quality)



When: Task is easy

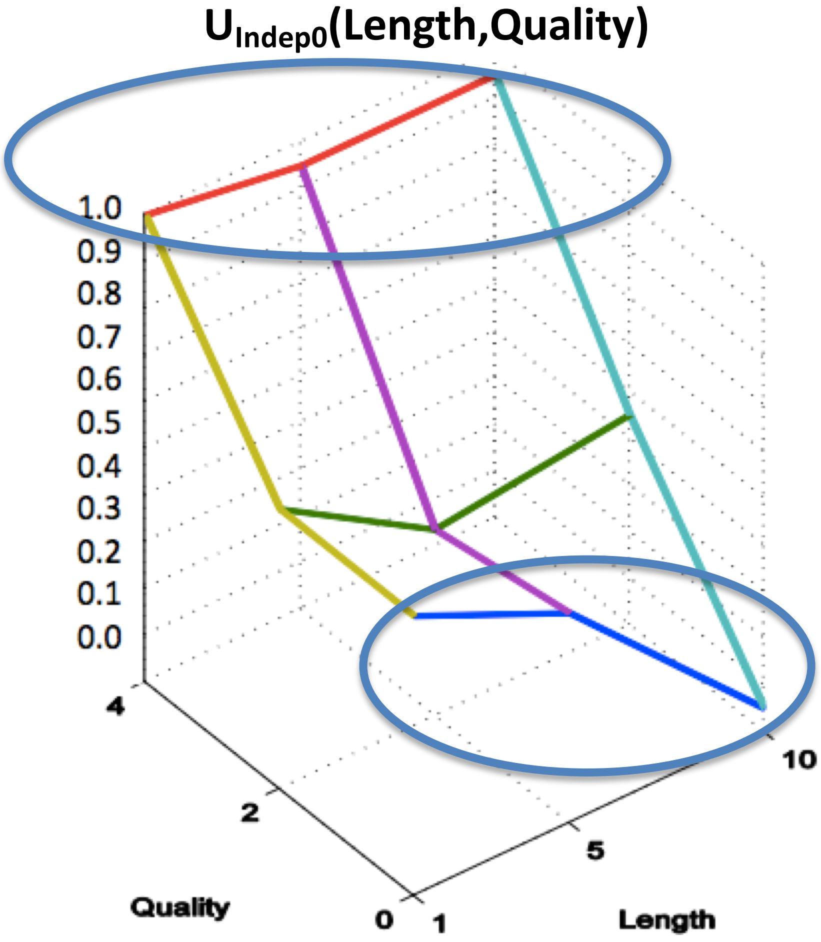
U_{Indep1}(Length,Quality)



- Non-decreasing in Q and non-increasing in L

User 11 – curvature of partial function

When: Task is difficult



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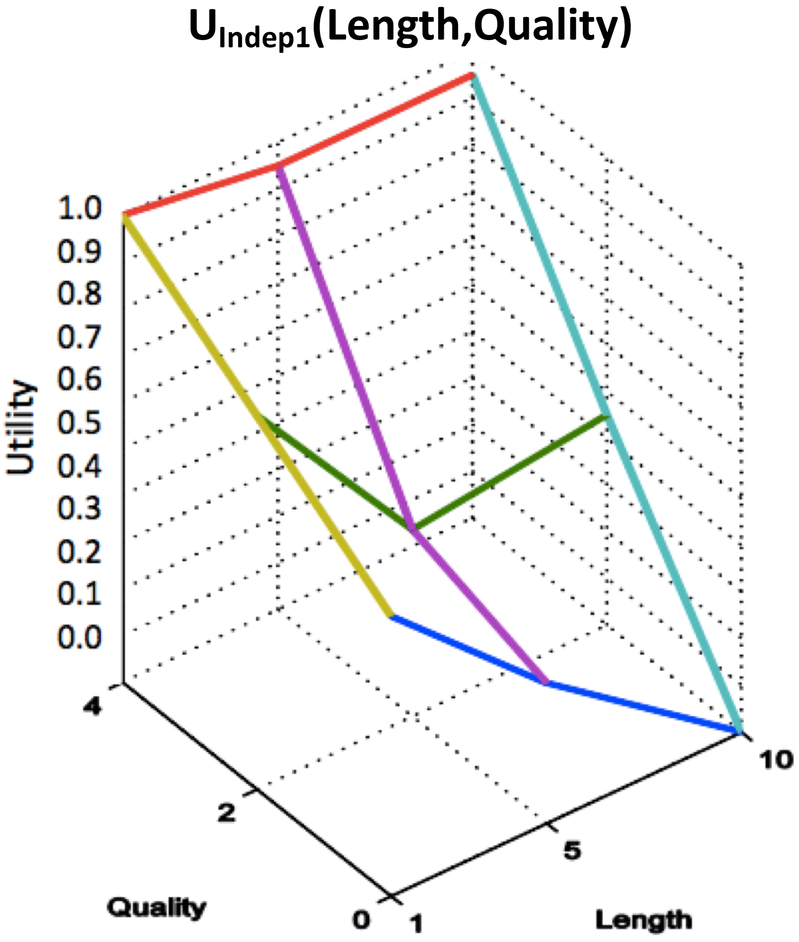

- Different structure as Q changes

Rest of the Study

- Imagine carrying out 50 highlighting tasks vs. doing 50 highlighting tasks

- Conceptual vs. experiential preferences
  - What you think you like isn’t what you actually like

- Adapting elicitation procedure to elicit experiential preferences

Key Ideas

- Main concepts
  - Eliciting a utility function via incremental constraints
  - Standard gamble query: What is *pr* s.t. $SG(pr) = o_i$?
  - Bound query: Given *pr,* is $SG(pr) > o_i$?
- Representation:
  - For simplicity, utility function may assume additive decomposition
- Algorithm:
  - Bound query for preference elicitation induces constraints for incremental refinement