Learning Analytics

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Recall BN Example #1

- Consider the following story:
  - If Bowen woke up too early (E), she needs caffeine (C)
  - If Bowen needs caffeine, she's likely to be grumpy (G)
  - If she is grump, then her lecture will not be as good (L)
  - If the lecture doesn't go smoothly, then students will be disappointed (S)

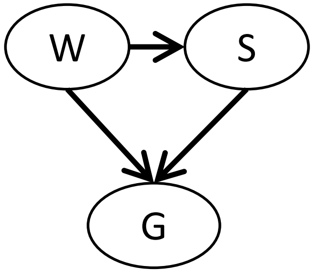
E = Woke up too early
C = need caffeine
G = gets grumpy
L = lecture not smooth
S = students disappointed

Recall BN Example #2

- Weather (W), Sprinkler (S), GrassWet (G)

Pr(W=sunny)	Pr(W=cloudy)	Pr(W=rainy)
0.6	0.3	0.1

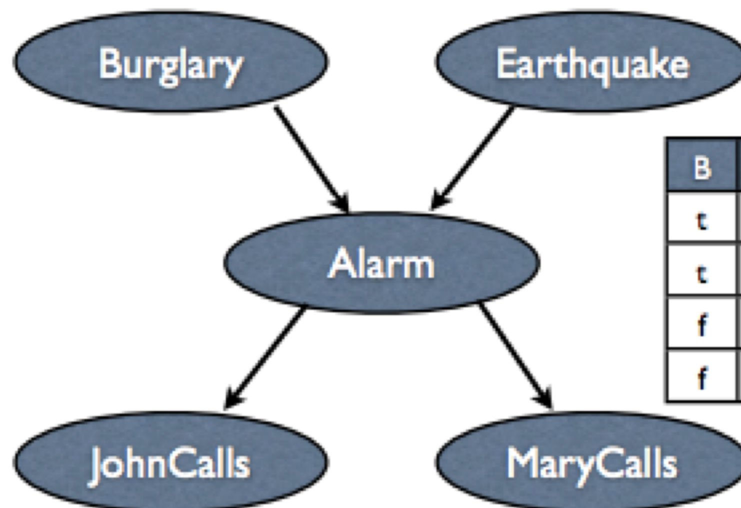
	Pr(S=on|W)	Pr(S=off|W)
W=sunny	0.1	0.9
W=cloudy	0.8	0.2
W=rainy	0.001	0.999



		Pr(G=wet|W,S)	Pr(G=dry|W,S)
W=sunny	S=on	0.9	0.1
W=sunny	S=off	0.001	0.999
W=cloudy	S=on	0.99	0.01
W=cloudy	S=off	0.2	0.8
W=rainy	S=on	1	0
W=rainy	S=off	0.9	0.1

BN Example #3

- Burglary network (from J. Pearl)
  - Burglary = burglary occurs at your house
  - Earthquake = earthquake occurs at your house
  - Alarm = alarm goes off
  - JohnCalls = John calls to report the alarm
  - MaryCalls = Mary calls to report the alarm



P(B)
0.001

P(E)
0.002

B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(JC|A)
t	0.90
f	0.05

A	P(MC|A)
t	0.70
f	0.01

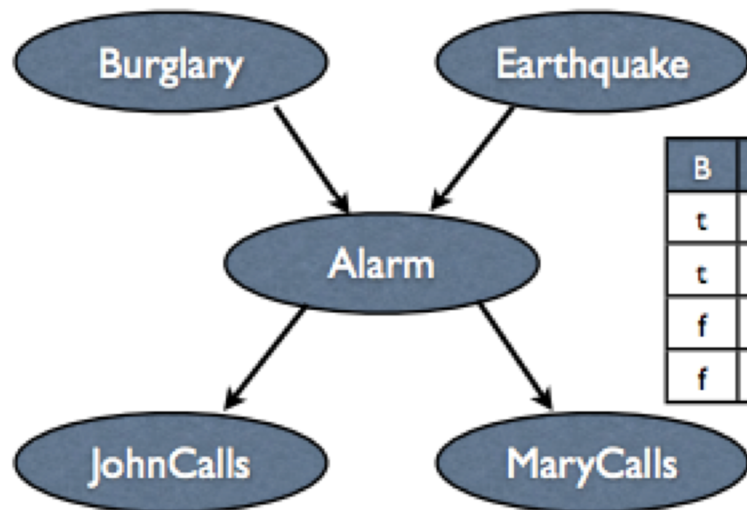
BN Example #3

- Compute an entry in the joint distribution:
  Pr(B=1, E=0, A=1, MC=1, JC=0)
  = ?

Expand and simplify
Based on BN independence assumptions

P(B)
0.001

P(E)
0.002



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t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(JC|A)
t	0.90
f	0.05

A	P(MC|A)
t	0.70
f	0.01

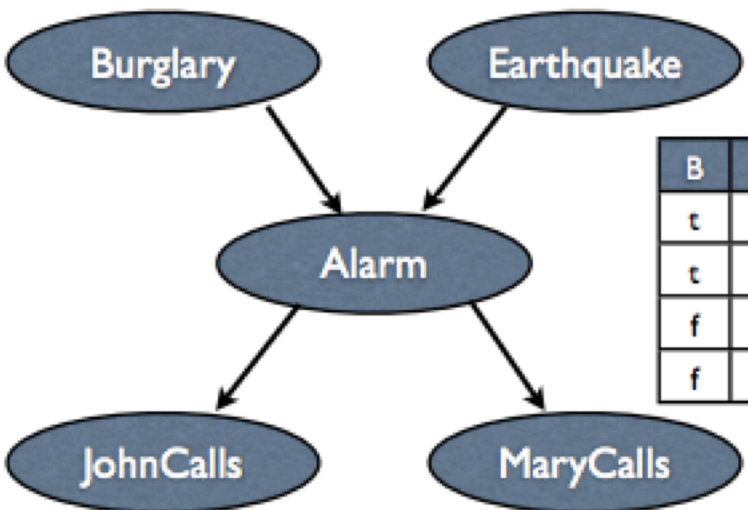
BN Example #3

- Compute the marginal probability that Mary calls
  Pr(MC=1) = ?

Rewrite via sum-out rule
Then simplify

P(B)
0.001

P(E)
0.002



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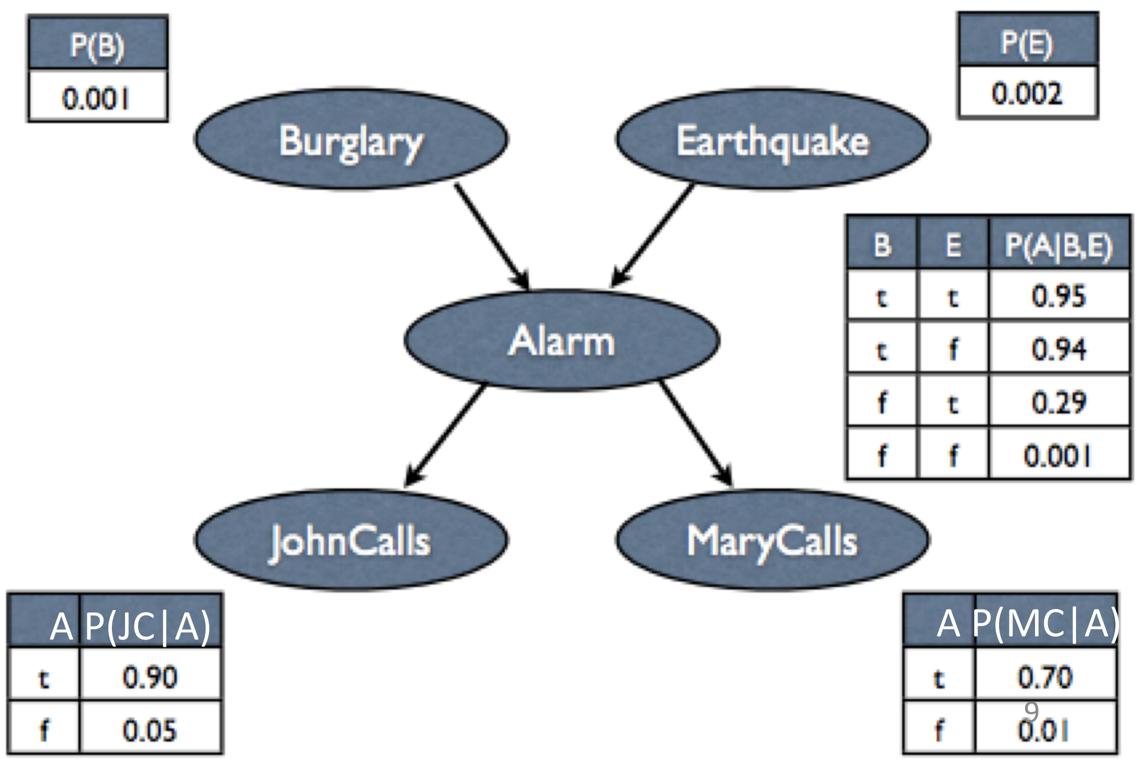
A	P(JC|A)
t	0.90
f	0.05

A	P(MC|A)
t	0.70
f	0.01

BN Example #3

- Making a prediction given evidence upstream in the graph
- E.g. Pr(MC=1 | B=1) = ?

Rewrite via conditional probability and sum-out rule, then simplify

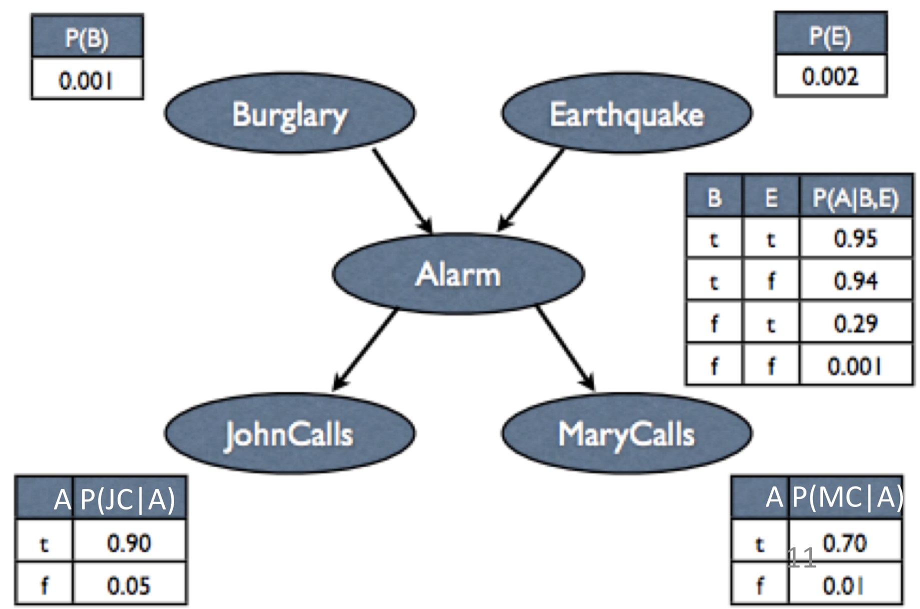


BN Example #3

- Finding an explanation given evidence downstream in the graph
- Pr(B=1 | MC=1) = ?
- Pr(E=1 | MC=1) = ?

Rewrite via Bayes rule

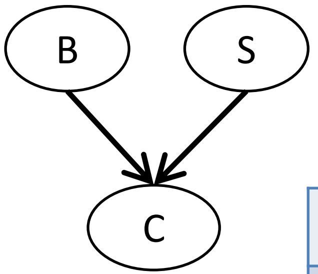
Then simplify



BN Example #4

- Suppose a college admits students who are either brainy (B) or sporty (S) or both.
- Let C denote the event that someone is admitted to college.
- Suppose in the general population that B and S are independent.

Pr(~B)	Pr(B)
0.5	0.5



Pr(~S)	Pr(S)
0.5	0.5

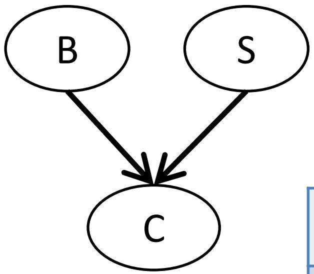
		Pr(~C|B,S)	Pr(C|B,S)
~B	~S	1	0
B	~S	0	1
~B	S	0	1
B	S	0	1

BN Example #4

- As it turns out, looking at population of college students (those for which C is observed to be true)
- It will be found that being brainy makes you less likely to be sporty and vice versa!
- Because either property alone is sufficient to explain the evidence on C

Pr(~S)	Pr(S)
0.5	0.5

Pr(~B)	Pr(B)
0.5	0.5

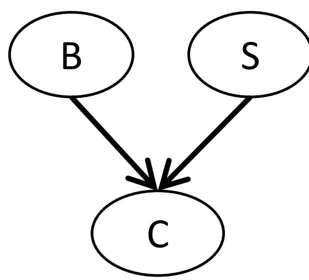


		Pr(~C|B,S)	Pr(C|B,S)
~B	~S	1	0
B	~S	0	1
~B	S	0	1
B	S	0	1

BN Example #4

- i.e. Pr(S=true | C=true, B=true) ≤ Pr(S=true | C=true) and Pr(B=true | C=true, S=true) ≤ Pr(B=true | C=true)
- This is known as the explaining away phenomenon
  - Happens when two causes “compete” to explain the observed data

Pr(~B)	Pr(B)
0.5	0.5



Pr(~S)	Pr(S)
0.5	0.5

		Pr(~C|B,S)	Pr(C|B,S)
~B	~S	1	0
B	~S	0	1
~B	S	0	1
B	S	0	1

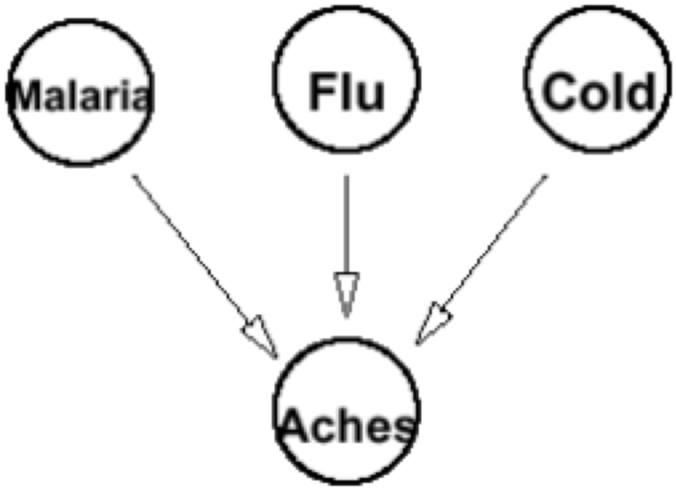
Constructing a Bayes Net

- Given any distribution over variables $X_1,\ldots,X_n$, we can construct a BN that faithfully represents that distribution

- Procedure is simple
  - Works with arbitrary orderings of variable set
  - But some orderings are much better than others!
  - Generally, if ordering/dependence structure reflects causal intuitions, a more natural, compact BN results

Causal Intuitions

- In this BN, we've used the ordering:
  Malaria, Cold, Flu, Aches
  to build the BN for distribution *P* for Aches
  - Variables can only have parents that come earlier in the ordering



How many parameters needed?

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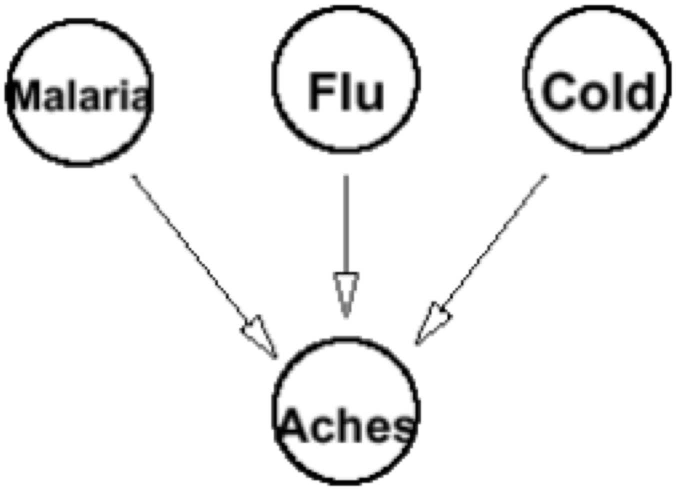
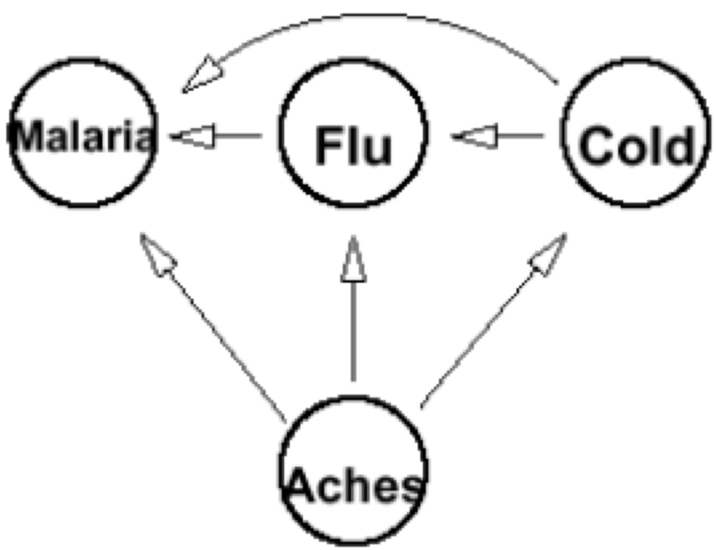


How many parameters needed?

- Top CPTs has $1 = 2^0$ numbers each
- Aches CPT has $8 = 2^3$ numbers

Causal Intuitions

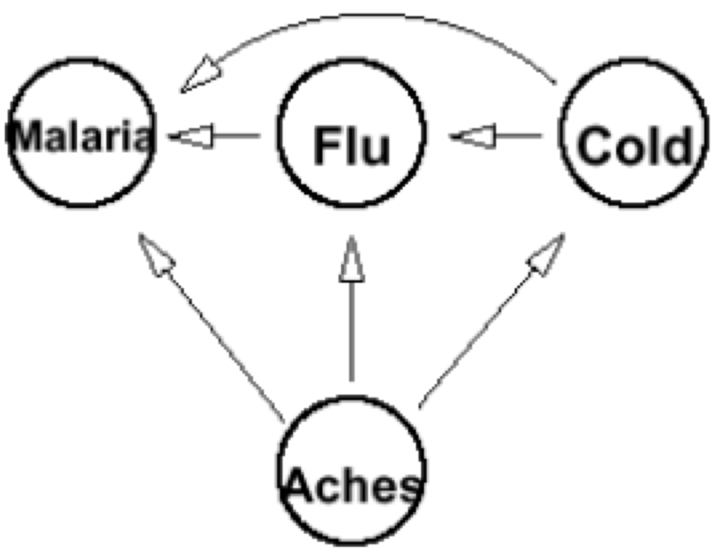
- Suppose we build the BN for distribution *P* using the opposite ordering:
  Aches, Cold, Flu, Malaria
  resulting network is more complicated!



- Malaria depends on Aches;
  Malaria depends on
  Flu, Cold given Aches
- Flu depends on Aches;
  Flu depends on Cold given Aches
- Cold depends on Aches

Causal Intuitions

- Suppose we build the BN for distribution *P* using the opposite ordering:
  Aches, Cold, Flu, Malaria
  resulting network is more complicated!



How many parameters needed?

- Malaria CPT has 8 = 2^3 numbers
- Flu CPT has 4 = 2^2 numbers
- Cold CPT has 2 = 2^1 numbers
- Aches CPT has 1 = 2^0 number

Group Exercise

- In groups of 2 or 3, build a BN (structure only) for the following scenario:
  - Model a student's exam performance based on various factors such as sleep, health, amount of study, difficulty of exam, mood, level of understanding, amount of practice, number of friends the student has, other exams around the same time, distractions, etc.
  - Identify 6-10 variables in your BN
  - Explain how your variables are defined and discretized
  - Use examples to justify the causal influences in your network

Group Exercise Part 2

- Pair 2-3 groups together
- Compare and contrast your BNs
- Argue + arrive at one BN
- Document and justify the causal influences in your network
- Review examples as a class

Key Ideas

- Main concept
  - Evidence downstream: you’re making prediction
  - Evidence upstream: you’re finding explanation
  - Inference with evidence upstream/downstream can be rewritten via the basic probability rules
- Representation:
  - Explaining away happens when multiple causes compete to explain observations
  - Multiple models to represent the same underlying RVs and causal influences