COSC 121: Computer Programming II

Dr. Bowen Hui

University of British Columbia Okanagan

Improved Sorting

- Insertion and selection sort algorithms:
  - Relatively easy to understand
  - Order N^2, given N items to sort
  - Not efficient when N is large

- Idea: Take advantage of the "halfing" technique used in binary search

- Faster algorithms that run in worst case of order N log N
  - Merge Sort
  - Quick Sort

General Idea

- Based on a recursive process: divide and conquer
- Merge sort: Given an array to sort:
  - Split in half
  - Each half being sorted separately
  - Results are merged together
- When to stop? (base case)
- To sort each half, the algorithm repeats the same process

Recursion vs. Iteration

- Iteration:
  - Loop takes one item at a time
  - Applies common set of commands

- Recursion:
  - Subdivides problem into smaller problems
  - Applies common set of commands to each subproblem
  - More in Ch. 12 of text

Recursive Thinking

- Consider the following list of numbers:

```
24, 88, 40, 37
```

- Such a list can be defined as follows:

```
A List is a:  number
        or a:  number  comma  List
```

- The concept of a *List* is used to define itself
  - Requires a stopping criteria called base case

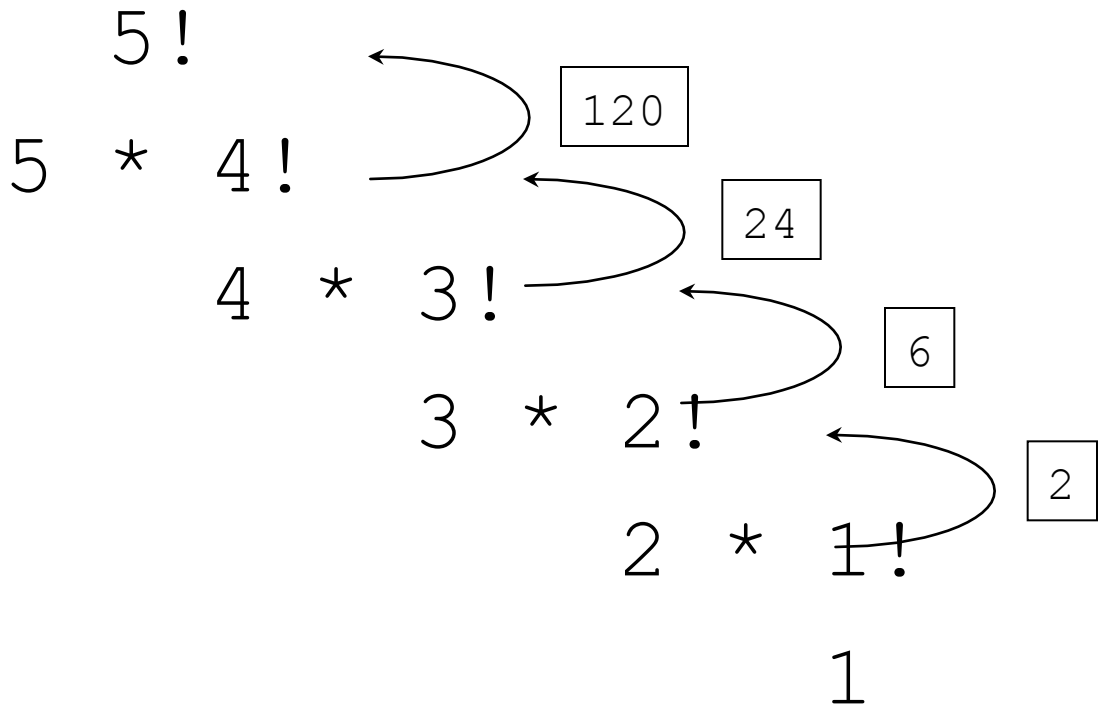
Recall: Factorial

- N! means for any positive integer N, this expression is defined to be the product of all integers between 1 and N inclusive
- Recursive definition:

```
1!   =   1
N!   =   N * (N-1)!
```

- A factorial is defined in terms of another factorial
- Eventually, get to base case of 1!

Recursive Factorial



Example

- Write a recursive definition of 5 * n, where n > 0.
  - 5 * 1 = 5
  - if n=2:
    5 * 2 = 5 + (5 * 1)
  - if n=3:
    5 * 3 = 5 + (5 * 2)
  - if n=4:
    5 * 4 = 5 + (5 * 3)
  - generally:
    5 * n = 5 + (5 * (n-1))

Recursion Exercise

- A recursive method is one that calls itself
- Example recursive method:

```
public int question( int x, int y )
{
   if( x == y )
     return 0;
   else
     return question(x-1, y);
 }
```

- What would `question(5, 3)` return?

Exercise 2

- Another example:

```
public int question( int x, int y )
{
   if( x == y )
     return 0;
   else
     return question(x-1, y) + 1;
 }
```

- What would `question(8, 3)` return?

Writing Recursive Methods

- Consider sum of 1 to any positive integer N
- Recursive definition:

$$\sum_{i=1}^{N} i = N + \sum_{i=1}^{N-1} i = N + N-1 + \sum_{i=1}^{N-2} i$$

$$= N + N-1 + N-2 + \sum_{i=1}^{N-3} i$$

$$\vdots$$

$$= N + N-1 + N-2 + \cdots + 2 + 1$$

Implementing Sum of 1 to N

```
// computes the sum of 1 to num
public int sum( int num )
{
    int result;

    // base case
    if( num == 1 )
        result = 1;
    else
        // general case
        result = num + sum( num-1 );

    return result;
}
```

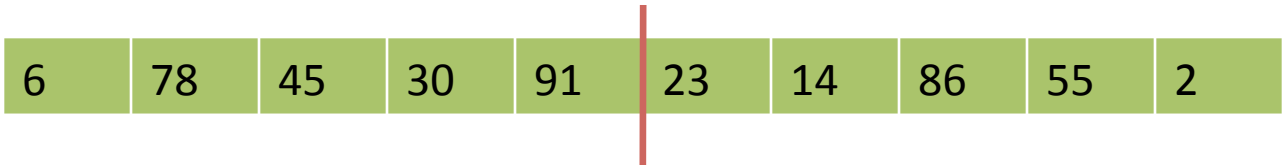
Recursion in Sorting

- Recursion is used to:
  - Take entire input (overall problem)
  - Subdivide input into smaller portions (subproblems)
  - Sort each smaller subproblem in the same way
  - Stop when base case is reached

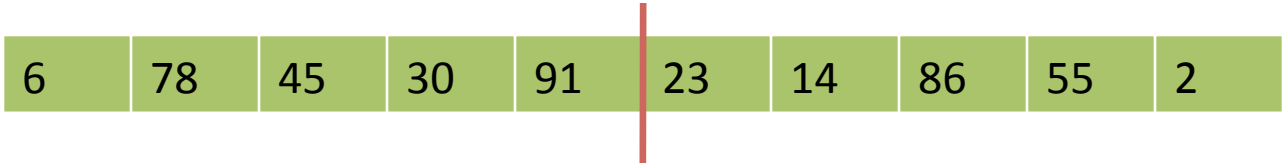
Merge Sort Illustration

6	78	45	30	91	23	14	86	55	2

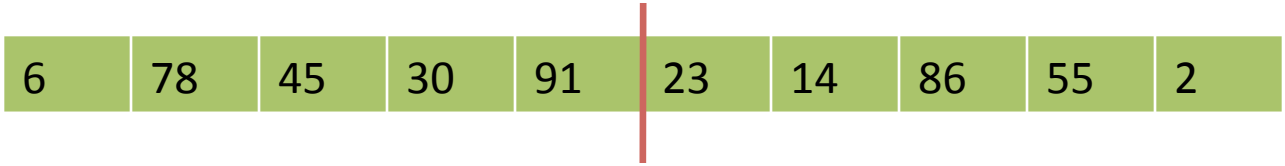
Merge Sort Illustration



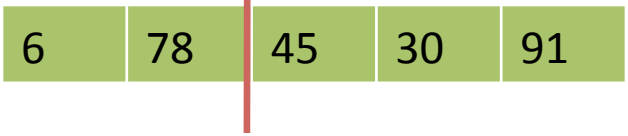
Merge Sort Illustration



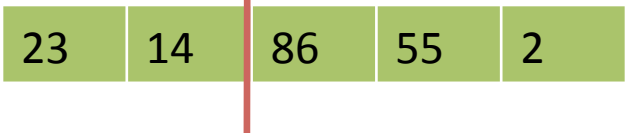
Merge Sort Illustration



split



split



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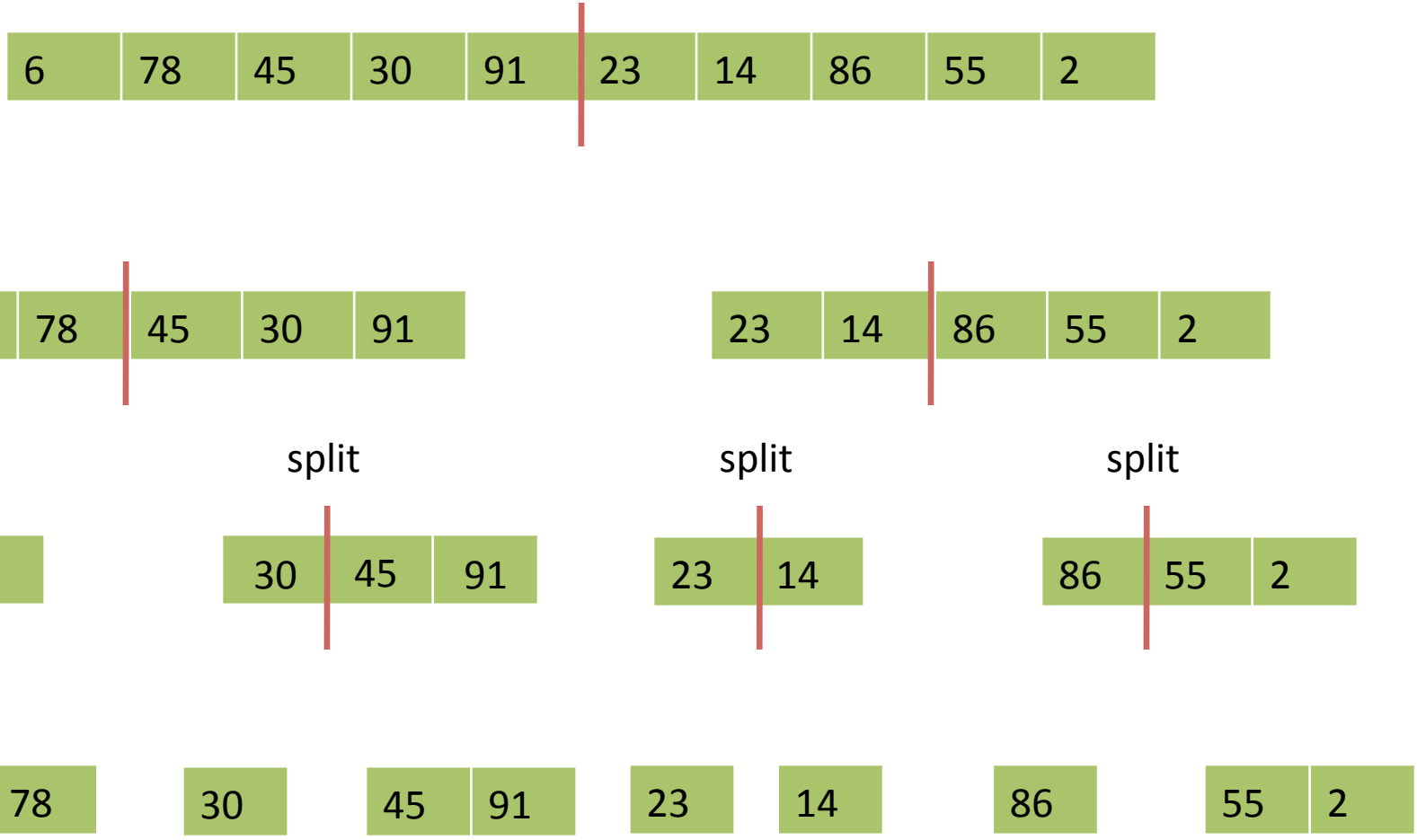

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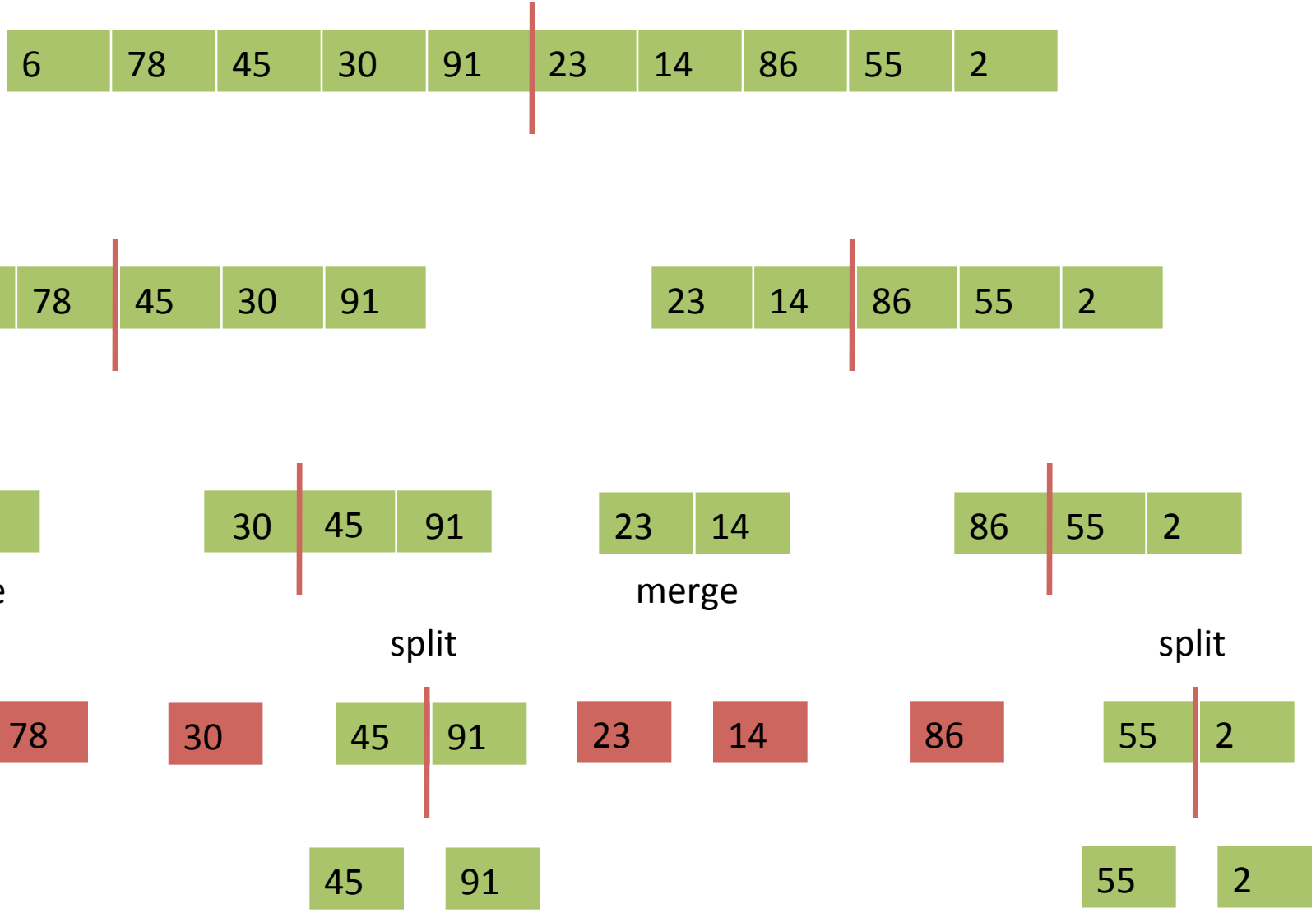

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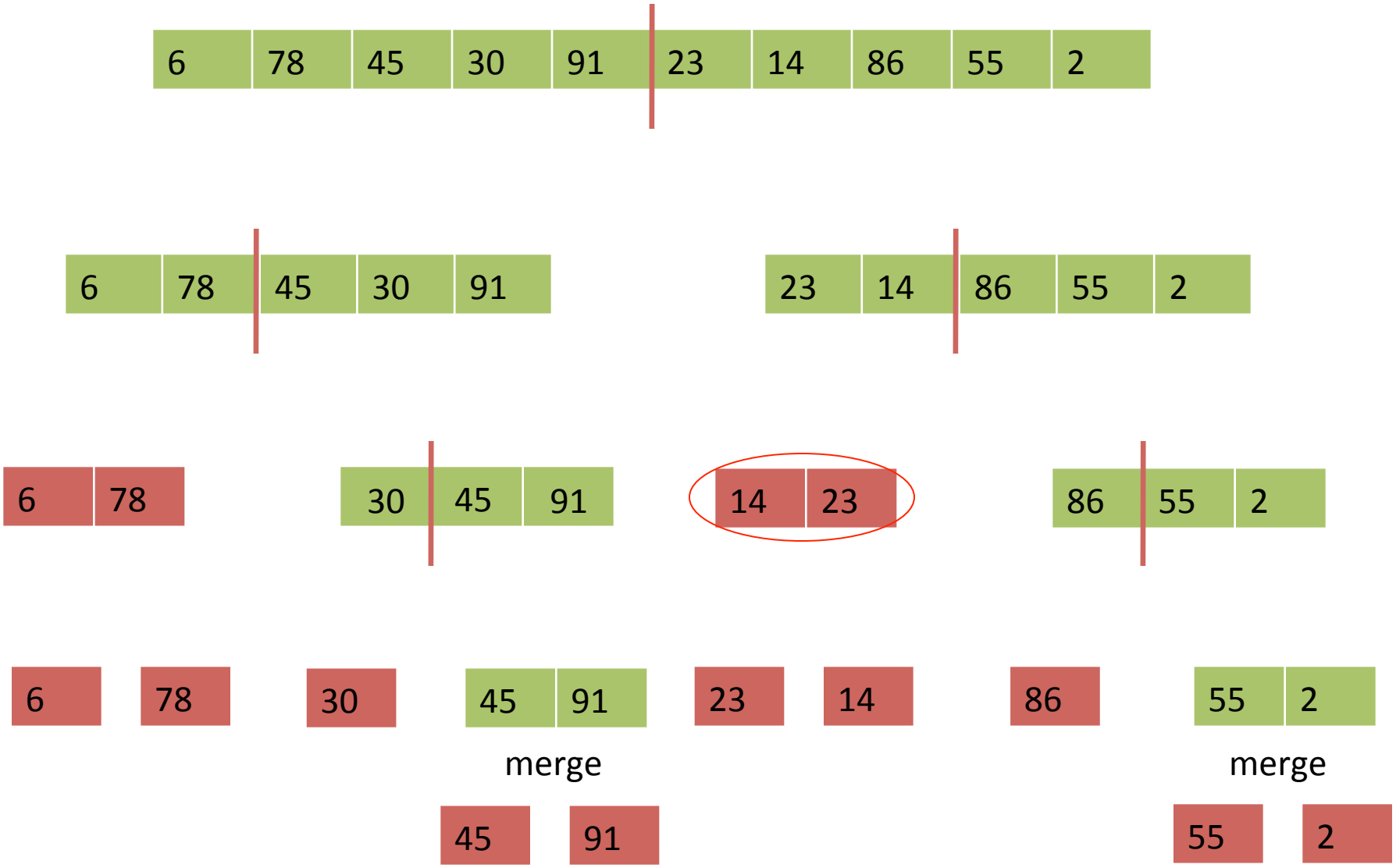
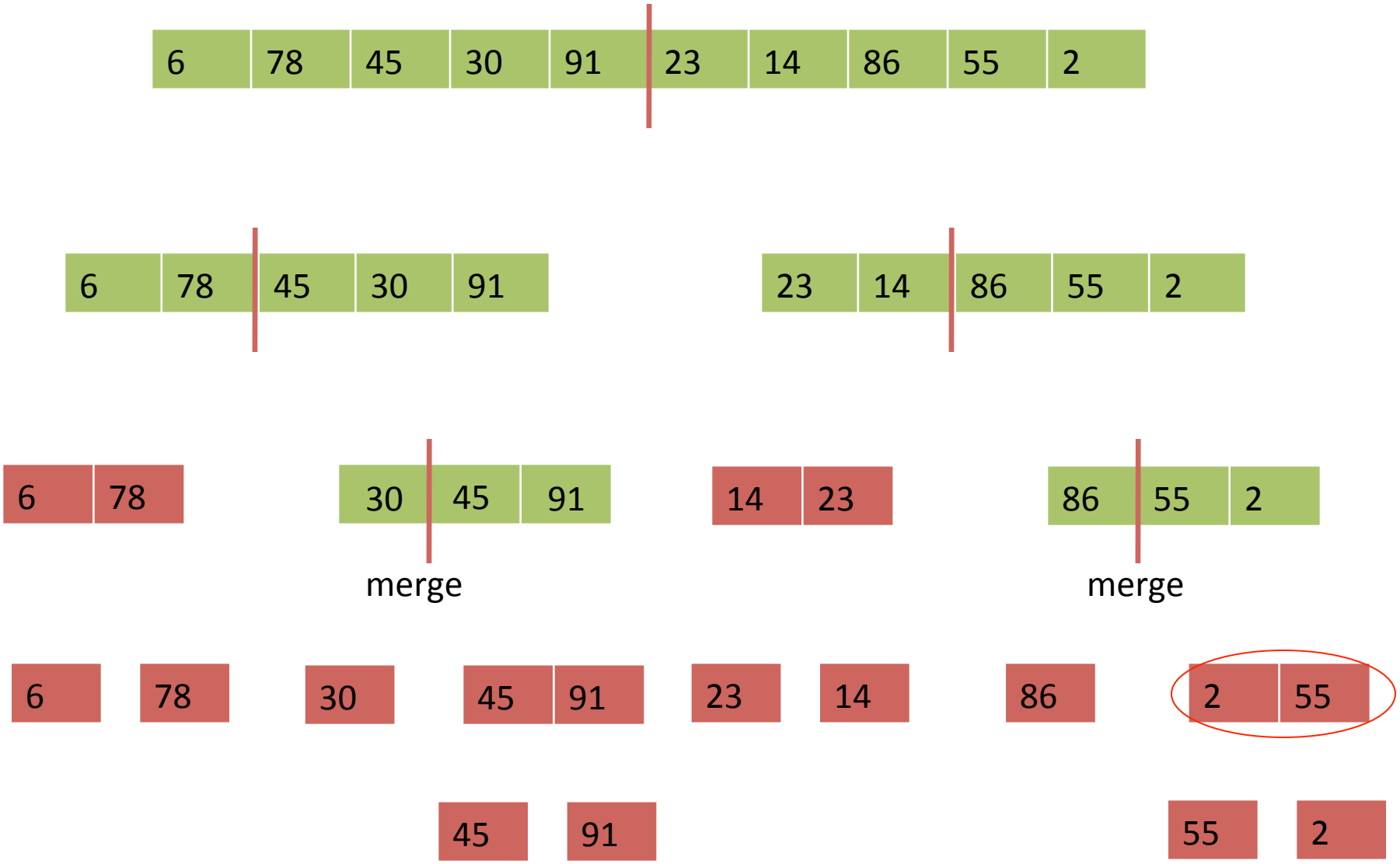


Merge Sort Illustration



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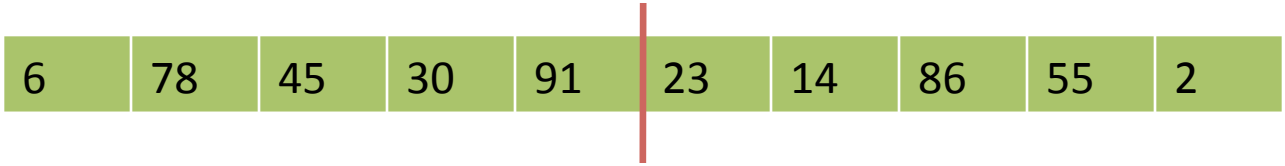

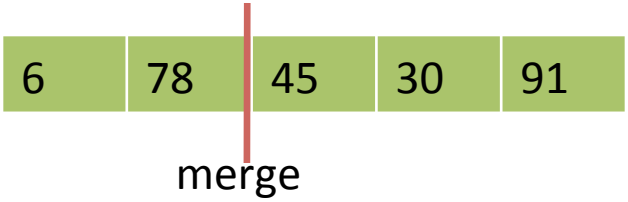

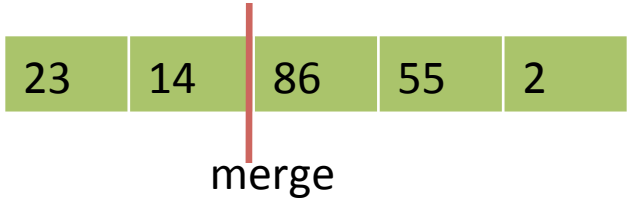

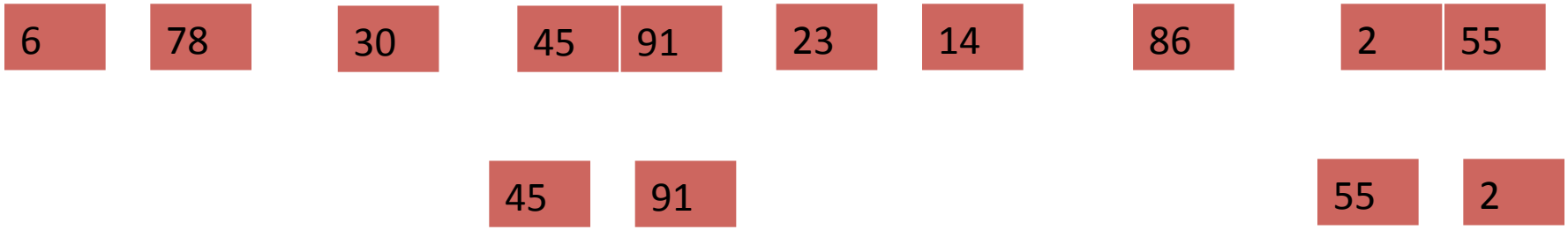

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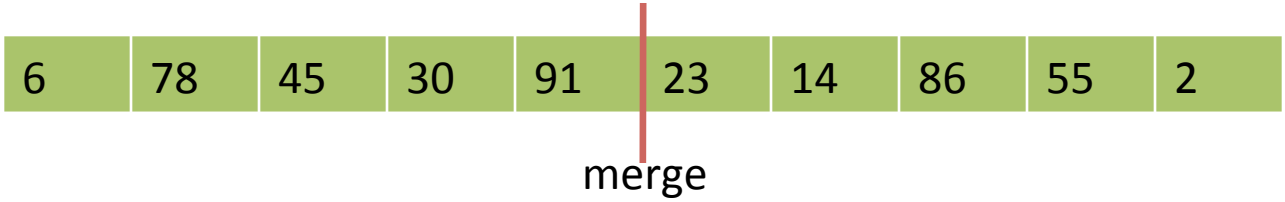

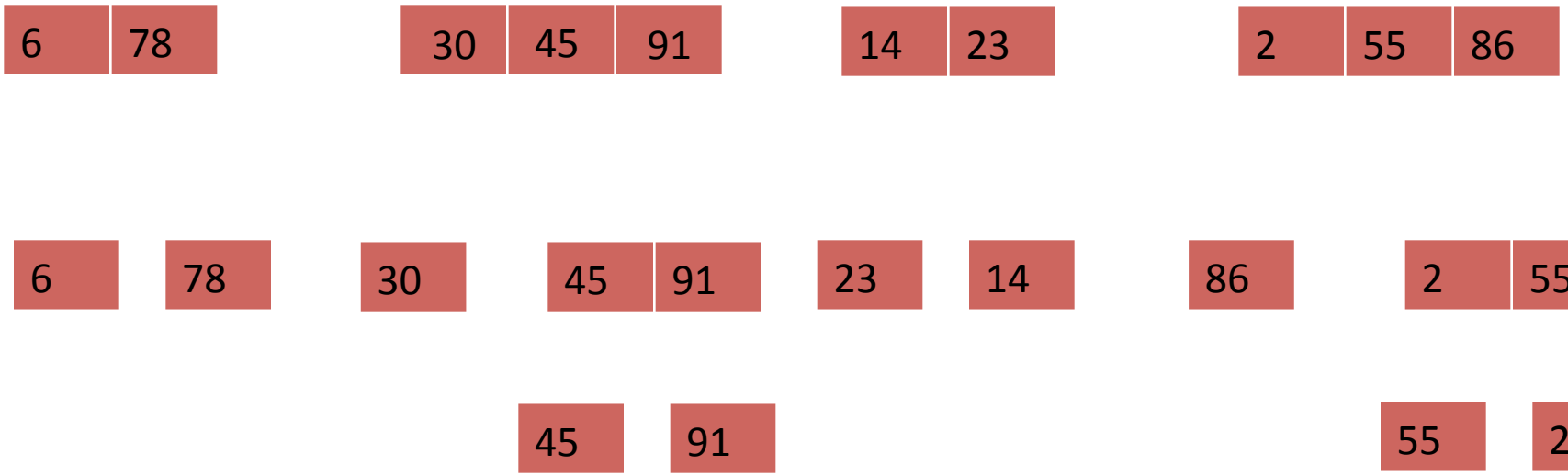

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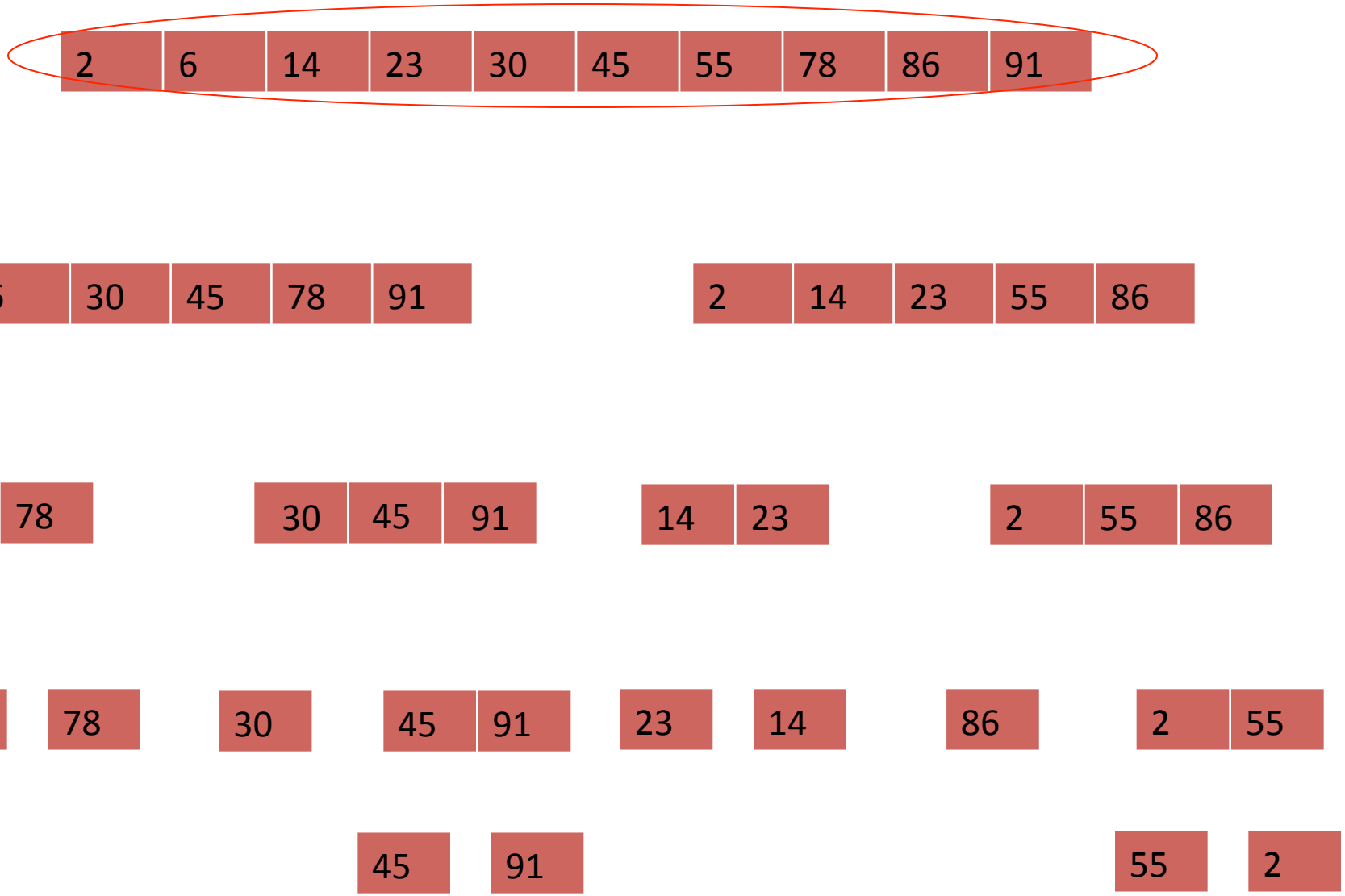

Exercise

56	13	1	99	65	12	27	8	44	83	2

Sort the following array using:

1. Merge sort

MergeSort on Integer array

```
public class MergeDemo
{
   public static void main (String[] args)
   {
      int[] someNumbers = {75, 9, 39, 42, 61, 21, 56, 32};

      for( int x : someNumbers )
         System.out.println( x );

      MergeSort sortObject = new MergeSort( someNumbers );

      sortObject.sort();

      for( int x : sortObject.getNumbers() )
         System.out.println( x );
   }
}
```

MergeSort on Integer array

```
public class MergeDemo
{
   public static void main (String[] args)
   {
      int[] someNumbers = {75, 9, 39, 42, 61, 21, 56, 32};

      for( int x : someNumbers )
         System.out.println( x );

      MergeSort sortObject = new MergeSort( someNumbers );

      sortObject.sort();

      for( int x : sortObject.getNumbers() )
         System.out.println( x );
   }
}
```

MergeSort.java

```
public class MergeSort
{
  // attribute
  private int[] numbers;

  // methods
  public MergeSort( int[] list ) {...}
  public void sort() {...}
  // other methods as needed...
}
```

Recursive Structure

```
public void sort()
{
   // if one elem left, stop: sorted
   // else:
   // sort on left half
   // sort on right half
   // combine two halves
}
```

Define Parameters for Recursion

```
public void sort()
{
  helper( 0, numbers.length-1 );
}

private void helper( int min, int max )
{
  // if one elem left, stop: sorted
  // else:
  // sort on left half
  // sort on right half
  // combine two halves
}
```

```
// main method to call for sorting
public void sort()
{
  helper( 0, numbers.length-1 );
}

// recursive method for partitioning the array into two halves
private void helper( int min, int max )
{
  System.out.println( "inside helper:" );
  System.out.println( "\tmin=" + min + ", max=" + max );
  if( min < max )
  {
    int mid = min + (max-min)/2;

    // sort each half independently
    helper( min,   mid );
    helper( mid+1, max );

    // when done, combine two sorted arrays
    combine( min, mid, max );
  }
  // otherwise, array is sorted
}
```

recursive method

method to do all the hard work

Main Steps in `combine()`

```
private void combine( int low, int mid, int high)
{
  // 1. create extra array called temp
  int[] temp = . . .

  // 2. setup indices
  int left  = . . .    // left side starts at low
  int right = . . .    // right side starts at mid+1
  int pos   = . . .    // pos (to merge) starts at low

  // 3. compare each value in left + right
  // copy smaller value into numbers[pos]
  while( left <= mid && right <= high ) { ... }

  // 4. copy rest of left side into numbers
  while( left <= mid ) { ... }
}
```

```
private void combine( int low, int mid, int high)
{
  // 1. create extra array called temp
  int[] temp = new int[numbers.length];
  for( int i=low; i<=high; i++ )
    temp[i] = numbers[i];

  // 2. setup indices
  int left  = low;      // left side starts at low
  int right = mid + 1;  // right side starts at mid+1
  int pos   = low;      // pos (to merge) starts at low

  // 3. compare each value in left + right
  // copy smaller value into numbers[pos]
  while( left <= mid && right <= high ) { ... }

  // 4. copy rest of left side into numbers
  while( left <= mid ) { ... }
}
```

```
private void combine( int low, int mid, int high)
{
  // 1. create extra array called temp
  // 2. setup indices
  // 3. compare each value in left + right
  // copy smaller value into numbers[pos]
  while( left <= mid && right <= high ) {
    if( temp[left] <= temp[right] ) {
      numbers[pos] = temp[left];
      left++;
    }
    else {
      numbers[pos] = temp[right];
      right++;
    }
    pos++;
  }
  // 4. copy rest of left side into numbers
 while( left <= mid ) { ... }
}
```

when do we exit this loop?

what was in numbers all this time?

```
private void combine( int low, int mid, int high)
{
  // 1. create extra array called temp

  // 2. setup indices

  // 3. compare each value in left + right
  // copy smaller value into numbers[pos]
  while( left <= mid && right <= high ) { ... }

  // 4. copy rest of left side into numbers
  while( left <= mid )
  {
    numbers[pos] = temp[left];
    left++;
    pos++;
  }
}
```

Tracing through MergeSort

- numbers = [75 9 39 42 61 21 56 32]

  0 1 2 3 4 5 6 7

- Method calls:

helper(0,7)

Tracing through MergeSort

- numbers = [75 9 39 42 61 21 56 32]

  0 1 2 3 4 5 6 7

- Method calls:

helper(0,7)

  helper(0,3)

Tracing through MergeSort

- numbers = [75 9 39 42 61 21 56 32]

  0 1 2 3 4 5 6 7

- Method calls:

helper(0,7)

  helper(0,3)

    helper(0,1)

Tracing through MergeSort

- numbers = [75 9 39 42 61 21 56 32]

  0 1 2 3 4 5 6 7

- Method calls:

helper(0,7)

  helper(0,3)

    helper(0,1)

      helper(0,0) // 0 < 0 is false

Tracing through MergeSort

- numbers = [75 9 39 42 61 21 56 32]
  0 1 2 3 4 5 6 7
- Method calls:

```
helper(0,7)
   helper(0,3)
      helper(0,1)
         helper(0,0)          // 0 < 0 is false
         helper(1,1)          // 1 < 1 is false
```

Tracing through MergeSort

- numbers = [75 9 39 42 61 21 56 32]
  0 1 2 3 4 5 6 7
- Method calls:

helper(0,7)
  helper(0,3)
    helper(0,1)
      helper(0,0) // 0 < 0 is false
      helper(1,1) // 1 < 1 is false
      combine(0,0,1)
        temp = [75 9]
          0 1

Tracing through MergeSort

- numbers = [75 9 39 42 61 21 56 32]
  0 1 2 3 4 5 6 7
- Method calls:

helper(0,7)
  helper(0,3)
    helper(0,1)
      helper(0,0) // 0 < 0 is false
      helper(1,1) // 1 < 1 is false
      combine(0,0,1)
        temp = [75 9]
          0 1 left=0, right=1, pos=0
- numbers = [9 9 39 42 61 21 56 32] when exit comparison loop
  0 1 2 3 4 5 6 7

Tracing through MergeSort

- numbers = [75 9 39 42 61 21 56 32]
  0 1 2 3 4 5 6 7
- Method calls:

helper(0,7)
  helper(0,3)
    helper(0,1)
      helper(0,0) // 0 < 0 is false
      helper(1,1) // 1 < 1 is false
      combine(0,0,1)
        temp = [75 9]
          0 1 left=0, right=1, pos=0
- numbers = [9 9 39 42 61 21 56 32] when exit comparison loop
  0 1 2 3 4 5 6 7
- numbers = [9 75 39 42 61 21 56 32] when exit combine
  0 1 2 3 4 5 6 7

Merge Sort

- Very intuitive and easy to remember
- New technique: recursion
- Sort method:
  - Manages recursive structure
- Combine method:
  - Keeping track of indices to sort in-place
- Next:
  - Continue tracing implementation + quicksort