Two non-linear generalizations of Neumann's Lemma

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In this talk I will present new invertibility theorems for non-necessary linear operators on a real Banach space, similar to the linear Cazassa-Christenses Lemma. Such invertibility results go back to the so-called Neumann Lemma.

This Lemma has various well-known consequences including two classical theorems:

- the set of all the invertible linear bounded operator defined on a real Banach space is open;
- the function which assigns to each linear bounded operator defined on a real Banach space its spectral radius is upper semi-continuous.

Our analysis benefits from the introduction of a new concept, strongly related to the Birkhoff-James orthogonality, that is the notion of near operator introduced at the end of the eighties by Sergio Campanato in a series of papers, through which he was able to unify existence and regularity results for PDE's and systems in non-divergence based on the Schauder method, the Lax-Milgram theorem, the Cordes theorem and the theory of monotone operators.

This talk summarizes a joint work with Emil Ernst and Annamaria Barbagallo.