Quasidense sets and multifunctions

Abstract

Let E be a nonzero real Banach space. “Quasidensity” is a concept that can be applied to subsets of $E \times E^*$ (or equivalently to multifunctions from E into E^*). Every (possibly nonmonotone) quasidense set is of “type (NI)”. Every closed quasidense monotone set is maximally monotone, but there exist maximally monotone sets that are not quasidense. The graph of the subdifferential of a proper, convex lower semicontinuous function on E is quasidense. The graphs of certain subdifferentials of certain nonconvex functions are also quasidense. (This follows from joint work with Xianfu Wang.) We discuss the “coincidence sets” of certain convex functions. The closed monotone quasidense sets enjoy a sum theorem and a parallel sum theorem. We know of ten conditions equivalent to the statement that a closed monotone set be quasidense, but quasidensity seems to be the only one of the ten that extends easily to nonmonotone sets. We give examples in general Banach spaces, Hilbert spaces and finite dimensional spaces.